A Scalable Algorithm for the Optimal Trajectory of a Massive Swarm of UAV Base Stations Using Lagrangian Mechanics

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Overview

Introduction

Some UAV applications in wireless networks :

- Replace infrastructure in an ad hoc manner
- Relay signal to reach out of coverage users
- Disseminate or collect data from ground stations/sensors
- Offload cellular network in highly crowded areas

In this presentation :

- We focus on offloading/data collection
- We consider the path planning problem
- We rely on the Lagrangian mechanics framework

Related Work

On the optimal trajectories of *communicating* UAVs :

- Discretize time and space and formulate a mixed integer non-convex optimization problem, e.g. [Meng et al., 2022, Zema et al., 2022, Liu et al., 2022] ⇒ flexible but complex (not more than 10 UAVs)
- Learn trajectory from previous experience with RL, see [Bayerlein et al., 2021, Moon et al., 2021] ⇒ scalability issues and scenario-specific
- Solutions based on HJB equations [Coupechoux et al., 2023, Chai and Lau, 2021] ⇒ not more than 30 UAVs

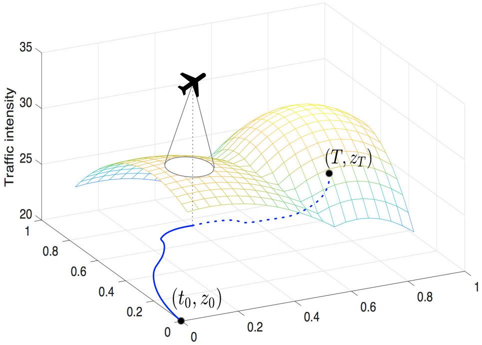
Direct and indirect methods in aircraft trajectory planning :

- Direct methods : time, locations and controls are discretised leading to non-linear programs like in [Wu et al., 2018]
- Indirect methods : analytical solutions from the calculus of variations and first order necessary conditions ⇒ our approach to deal with thousands of UAVs

On Lagrangian Mechanics

- Lagrangian mechanics describes the trajectory of a particle.
- It is based on a quantity $\mathcal{L}$ called *Lagrangian*, which is an energy and has a standard form $\mathcal{L} = T - V$
- T is the kinetic energy and V is the potential energy
- We define the action $S = \int_{t_0}^{T} \mathcal{L} dt$
- Hamilton's principles states that the trajectory followed by the particle optimises the action
- By expressing this principle as a variational principle, we get a set of differential equations called Euler-Lagrange equations needed to calculate the optimal trajectory

System Model



- There N UAVs and a single hotspot at location z_h around which the traffic to be served is distributed
- UAV k starts at $z_k^0 \triangleq z_k(t_0)$ at t_0 and shall reach a destination $z_k^T \triangleq z_k(T)$ at T.
- We want to optimize the trajectories in terms of energy consumption and served traffic.

Problem Formulation

- The running cost ($\sim$ Lagrangian) :

$$\mathcal{L}(t, Z, A) = \sum_{i=1}^{N} \left(\frac{K}{2} \|a_i\|^2 - \frac{1}{2} u_0 \|z_i - z_h\|^2 \right) + v_0 \sum_{\substack{i<j \\ i,j=1:N}} \|z_i - z_j\|^2 \quad (1)$$

where $Z = (z_1, z_2 \ldots z_N)$ and $A = (a_1, a_2 \ldots a_N) = \dot{Z}$

1st term is the kinetic energy, related to velocity $a_i = \dot{z}_i$;
2nd term is a traffic intensity around a hotspot z_h ($\sim$ potential energy)
3rd term is coupling cost (we don't want UAVs to be too close from each other)
v_0 should be negative if we want to avoid collisions

- In compact form :

$$\mathcal{L}(t, Z, A) = \frac{K}{2} \left(|||A|||^2 + \omega^2 \; ||| Z - Z_h |||^2 + \epsilon \Omega^2 \phi(Z) \right) \quad (2)$$

Single UAV Case : Main Result

See our previous work [Coupechoux et al., 2023] : from Euler-Lagrange, we derive the optimal trajectory

Lemma (Euler-Lagrange Equations)

Along the optimal trajectory $z^*(t)$ that starts from z_0 at t_0 and ends at z_T at T :

$$\frac{d}{dt}\nabla_a \mathcal{L}(t, z^*(t), a^*(t)) = \nabla_z \mathcal{L}(t, z^*(t), a^*(t)) \quad (3)$$

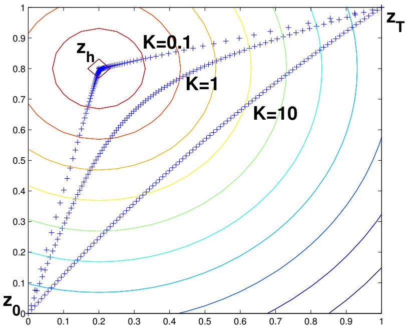
Theorem

If $u_0 < 0$, the optimal trajectory is

$$z^*(t) = \frac{z_T \sinh(\omega(t - t_0)) + z_0 \sinh(\omega(T - t))}{\sinh(\omega(T - t_0))} \quad (4)$$

where $\omega^2 = -\frac{u_0}{K}$.

Single UAV Case : Traffic Hot Spot $u_0 < 0$



- When K is small, the UAV goes fast to the hot spot, reduces its speed in the vicinity and then goes fast to the destination
- When K increases, the trajectory tends to the straight line

Multi-UAV Case : The Coupling Cost

Let's have a closer look at the coupling cost

$$\phi(Z) = \sum_{\substack{i<j \\ i,j=1:N}} \|z_i - z_j\|^2 = N \left(\sum_{i=1}^{N} \|z_i - g\|^2 \right) \quad (5)$$

where g is the barycentre of the UAVs.

- It is a quadratic form
- It can be diagonalized with eigenvalues and eigenvectors :
  - subspace H : $\lambda = 0 \rightarrow$ all $\chi(T) = (T, ..., T)$
  - subspace $H^{\perp}$: $\lambda = N \rightarrow$ all Z such that $g = 0$
- Any $\bar{Z}$ can be decomposed along H and $H^{\perp}$, let's take $\bar{Z} = Z - \chi(z_h)$

$$\bar{Z}(t) = \underbrace{\xi(t)}_{\in H^{\perp}} \oplus \underbrace{\gamma(t)}_{\in H} \quad (6)$$

where $\xi(t) = \bar{Z} - \chi(\bar{g})$, $\gamma(t) = \chi(\bar{g})$ and $\bar{g} = g - z_h$ (centered wrt hotspot)

Multi-UAV Case : Lagrangian Expression

Let's re-write the Lagrangian by using our decomposition along H and $H^{\perp}$:

$$
\begin{aligned}
\bar{\mathcal{L}} &= \frac{K}{2} \left[\|\dot{\bar{Z}}\|^2 + \omega^2 \ ||| \bar{Z} |||^2 + \epsilon \ \Omega^2 \ \phi(\bar{Z}) \right] \\
&= \frac{K}{2} \left[|||\dot{\xi}|||^2 + |||\dot{\gamma}|||^2 + \omega^2 \left(|||\xi|||^2 + |||\gamma|||^2 \right) + \epsilon \ \Omega^2 \left(N \times |||\xi|||^2 + \ 0 \times |||\gamma|||^2 \right) \right] \\
&= \frac{K}{2} \left[|||\dot{\xi}|||^2 + (\omega^2 + \epsilon N \Omega^2) \ ||| \xi |||^2 \right] + \frac{K}{2} \left[|||\dot{\gamma}|||^2 + \omega^2 \ ||| \gamma |||^2 \right] \qquad (7)
\end{aligned}
$$

Multi-UAV Case : Euler-Lagrange Equation

Gradients can be computed along every component H and $H^\perp$:

$$\begin{aligned} \nabla_{\bar{Z}} \bar{\mathcal{L}} &= \nabla_\xi \bar{\mathcal{L}} \oplus \nabla_\gamma \bar{\mathcal{L}} \\ &= K\left((\omega^2 + \epsilon N \Omega^2)\, \xi \oplus \omega^2 \gamma \right) \\ \nabla_A \bar{\mathcal{L}} &= \nabla_{\dot{\xi}} \bar{\mathcal{L}} \oplus \nabla_{\dot{\gamma}} \bar{\mathcal{L}} \\ &= K\left(\dot{\xi} \oplus \dot{\gamma} \right) \\ \rightarrow \frac{d}{dt} \nabla_A \bar{\mathcal{L}} &= K\left(\ddot{\xi} \oplus \ddot{\gamma} \right) \end{aligned}$$

Now Euler-Lagrange equations become :

$$\frac{d}{dt} \nabla_A \bar{\mathcal{L}} - \nabla_{\bar{Z}} \bar{\mathcal{L}} = K\Big(\big(\ddot{\xi} - (\omega^2 + \epsilon N \Omega^2)\, \xi\big) \oplus \big(\ddot{\gamma} - \omega^2 \gamma\big) \Big) = 0$$

We obtain two independent Euler-Lagrange equations !

$$\ddot{\xi} - (\omega^2 + \epsilon N \Omega^2)\, \xi = 0 \text{ and } \ddot{\gamma} - \omega^2 \gamma = 0 \tag{8}$$

Multi-UAV Case : Optimal Trajectories

How to construct optimal trajectories ?

1) First solve $\ddot{\gamma} - \omega^2 \gamma = 0$

- recall that $\gamma = \chi(g - z_h)$

$\Rightarrow$ the barycentre of the UAVs follows a *single UAV hyperbol*

2) Then solve $\ddot{\xi} - (\omega^2 + \epsilon N \Omega^2) \xi = 0$

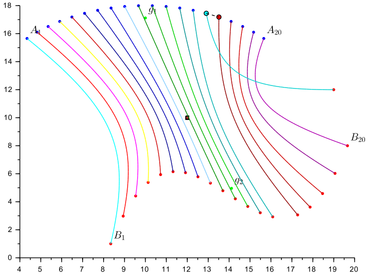
- recall that $\xi_k = z_k - g$ for UAV k

$\Rightarrow$ UAV k follows an hyperbol (> 0), an ellipse (< 0) or a line ($= 0$) *relatively to the barycentre* depending on the sign of $\omega^2 + \epsilon N \Omega^2$

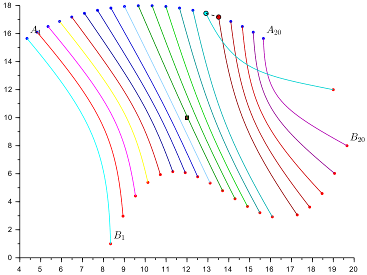
We obtain closed-form formulas for the trajectories !

Numerical Experiments : Coupling

No coupling



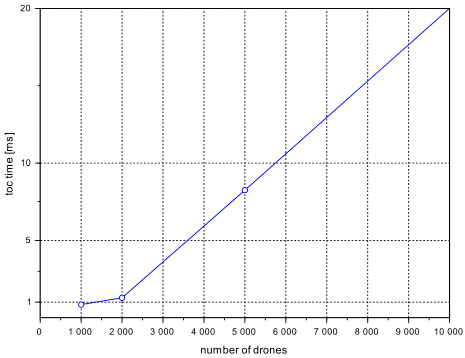
Repulsive coupling



- 20 UAVs optimal trajectories
- By playing with Ω, we can control the distance between the UAVs along their trajectories

Numerical Experiments : Scalability

Scalability : algorithm computing time vs. number of UAVs.



- Scilab 2023, commercial laptop

⇒ 10 000 coordinated trajectories in 20 ms

Conclusion

Some contributions :

- Problem formulation using Lagrangian mechanics that extends our previous work to swarms of UAVs
- A coupling parameter to control the distance between UAVs
- Closed-form expressions of the trajectories when there is a single hotspot
- A scalable solution : thousands of optimal coordinated trajectories obtained at a click speed

Some open points :

- What about multi-hotspots ?
- Can we go beyond quadratic ?
- How to incorporate details from more realistic models ?

Thank you for your attention !

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