Random Access

Marceau Coupechoux

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Outlines

- Aloha
- CSMA/CA

Pure Aloha

How does it work ?

- N (possibly infinite) stations transmit packets to a single receiver over a single shared channel
- Upon arrival of a new packet in some station's buffer, the packet is transmitted on the shared channel
- There is a collision if at least two transmissions overlap at the receiver, otherwise, the transmission is successful
- There is an immediate feedback from the receiver on a different channel to positively or negatively acknowledge the reception of a packet

Pure Aloha : Infinite Population Model I

Assumptions :

- (1) $N = \infty$
- (2) Packets have equal size, channel data rate is constant, let T be the transmission time of a packet, propagation is supposed to be negligible
- (3) Packets arrive according to a Poisson process of rate λ packets/s
- (4) Upon arrival of a packet, it is immediately transmitted on the channel
- (5) Stations transmit a single packet and die
- (6) In case of collision, packets are retransmitted so that the overall arrival process of packets is assumed to be Poisson of rate $g > \lambda$ packets/s

Recall that for a Poisson process of rate λ, the probability of having $k = 0, 1, \ldots$ arrivals in a time-interval of duration T is :

$$\mathbb{P}[k \text{ arrivals}] = \frac{(\lambda T)^k}{k!} e^{-\lambda T} \tag{1}$$

Pure Aloha : Infinite Population Model II

Analysis

- Consider a packet transmitted at time t. It is successful if no packet is transmitted in the interval $(t-T;t+T)$. $2T$ is called the *vunerable period*. This happens with probability P_{succ} of having no packet arrival in an interval of duration $2T$:

$$P_{succ} = e^{-2gT} \tag{2}$$

- Packets arrives at a rate of g per second. The rate of successful packets is gP_{succ}. When a packet is successful, the channel carries information during T. Otherwise, the channel does not carry any useful information. Let S be the fraction of time the channel carries useful information :

$$S = gTe^{-2gT} = Ge^{-2G} \tag{3}$$

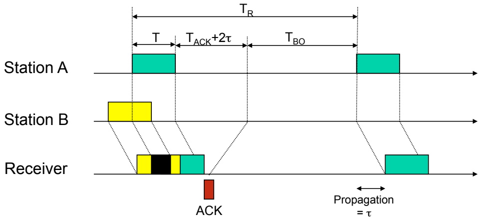
  where $G \triangleq gT$ is the *normalized offered load*

- S is maximized at $G = 1/2$ and $S^* = 1/2e \approx 0.18$

Pure Aloha : Average Delay I

Assumptions

- (1) W is the average delay between first packet transmission and its successful reception
- (2) We assume a one-way propagation delay of τ, a transmission duration of T_{ACK} for the ACK
- (3) Let T_{BO} be the average *backoff* time before retransmission. There is no limit on the number of retransmissions. We assume a random backoff duration uniformly drawn in the interval $[0, KT]$, where K is the backoff window length.



Pure Aloha : Average Delay II

Analysis

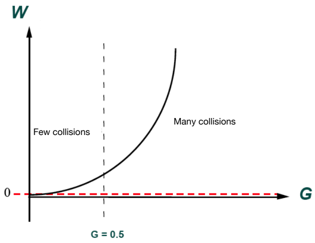
- We have :

$$W = T + \tau + R(T_{BO} + T + T_{ACK} + 2\tau) \tag{4}$$

where R is the average number of retransmissions and $T_{BO} = \frac{K+1}{2}T$

- We have :

$$R = \sum_{n=1}^{\infty} nP_{succ}(1 - P_{succ})^n = \frac{1 - P_{succ}}{P_{succ}} = e^{2G} - 1 \tag{5}$$



Slotted Aloha

How does it work ?

- N (possibly infinite) stations transmit packets to a single receiver over a single shared channel
- Stations are synchronized, time is slotted and slot duration equals the packet transmission durations
- Upon arrival of a new packet in some station's buffer, the packet is transmitted on the shared channel *at the next slot boundary*
- There is a collision if at least two transmissions occupy the same slot, otherwise, the transmission is successful
- There is an immediate feedback from the receiver on a different channel to positively or negatively acknowledge the reception of a packet

Slotted Aloha : Infinite Population Model I

Assumptions :

- (1) $N = \infty$
- (2) Packets have equal size, channel data rate is constant, let T be the transmission time of a packet, propagation is supposed to be negligible
- (3) Packets arrive according to a Poisson process of rate λ packets/s
- (4) Upon arrival of a packet, it is transmitted at the next slot boundary
- (5) Stations transmit a single packet and die
- (6) In case of collision, packets are retransmitted so that the overall arrival process of packets is assumed to be Poisson of rate $g > \lambda$ packets/s

Slotted Aloha : Infinite Population Model II

Analysis

- Consider a packet transmitted at time t (a slot boundary). It is successful if no packet is transmitted at t, i.e., if no packet has been generated in $(t - T; t)$. The vulnerable period has duration T. This happens with probability P_{succ} of having no packet arrival in an interval of duration T :

$$P_{succ} = e^{-gT} \tag{6}$$

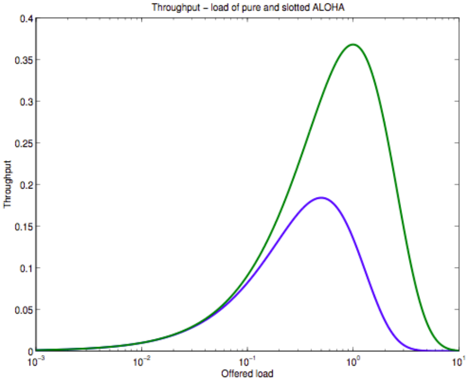
- Packets arrives at a rate of g per second. The rate of successful packets is gP_{succ}. When a packet is successful, the channel carries information during T. Otherwise, the channel does not carry any useful information. Let S be the fraction of time the channel carries useful information :

$$S = gTe^{-gT} = Ge^{-G} \tag{7}$$

  where $G \triangleq gT$ is the *normalized offered load*

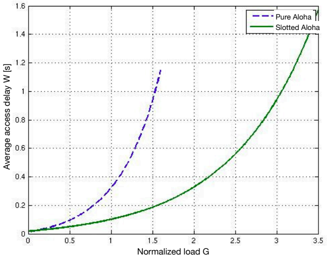
- S is maximized at $G = 1$ and $S^* = 1/e \approx 0.36$

Slotted Aloha : Infinite Population Model III



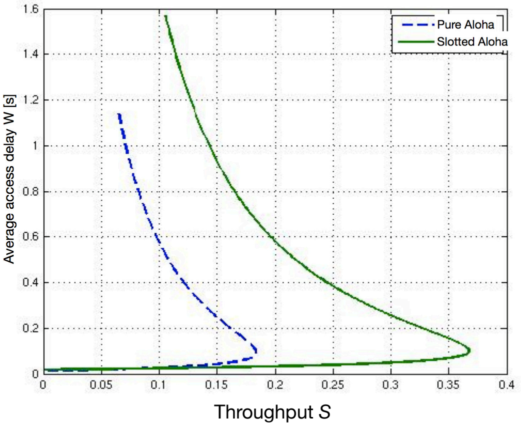
Slotted Aloha : Average Delay I

- Same assumptions as for pure Aloha, except that we have to wait in average $T/2$ between the packet generation and the next slot boundary, so that : $W = T + \tau + T/2 + R(T_{BO} + T + T_{ACK} + 2\tau)$
- We obtain : $W = (e^G - 1)((K+3)\frac{T}{2} + 2\tau + T_{ACK}) + \frac{3T}{2} + \tau$
- Take : Packet length = 1500 Bytes, ACK length = 40 Bytes, PHY data rate = 1 Mbps, $K = 5$, distance to receiver = 1000 m, we obtain :



Slotted Aloha : Average Delay II

Delay/throughput characteristic



Slotted Aloha : Finite Population Model I

Assumptions

- (1) $N < \infty$
- (2) Packets have equal size, channel data rate is constant, let T be the transmission time of a packet, propagation is supposed to be negligible
- (3) Packets arrive according to a Poisson process of rate λ/N packets/s at every station
- (4) Upon arrival of a packet, it is transmitted at the next slot boundary
- (5) Stations have a single packet buffer
- (6) In case of collision, packets are retransmitted so that the overall arrival process of packets is assumed to be Poisson of rate $g > \lambda$ packets/s
- (7) As an alternative to (3)-(6), stations transmit with probability p on a slot

Note : Assuming (3)-(6), the probability of transmission for a station on a slot is :

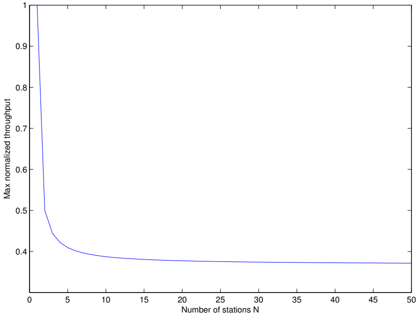
$$p = 1 - e^{-gT} \qquad (8)$$

so that assumptions (3)-(6) and (7) are equivalent.

Slotted Aloha : Finite Population Model II

Analysis

- On a given slot, a transmission is successful if there is a single transmission from one out of N stations, so that $S_N = Np(1-p)^{N-1}$
- S_N is maximized for $p = \frac{1}{N}$ and we obtain $S_N^* = (1 - \frac{1}{N})^{N-1} \rightarrow \frac{1}{e}$ when $N \rightarrow \infty$



Slotted Aloha : Stability Analysis I

Assumptions

- (1) $N < \infty$
- (2) Packets have equal size, channel data rate is constant, let T be the transmission time of a packet, propagation is supposed to be negligible
- (3) Packets arrive according to a Poisson process of rate λ/N at every station
- (4) In case of a collision, a packet has to be retransmitted by the transmitting station until the transmission is successful. A station with a packet to be retransmitted is said to be *backlogged*. A backlogged station retransmits with a fixed probability q_r
- (5) Stations have a single packet buffer : If packets arrive at a node while another packet is waiting for transmission or is colliding with other packets during its transmission, these new packets are discarded

Slotted Aloha : Stability Analysis II

Stability Analysis [2]

- The behavior of slotted Aloha can be modeled as a discrete time Markov chain, whose states are described by the number n of backlogged stations and which evolves at every time slot. Each of these stations transmits independently with probability q_r on a given slot
- Each of the remaining $N - n$ stations transmits with probability $q_a = 1 - e^{-\lambda T/N}$.
- Let $Q_a(i, n)$ be the probability that i *unbacklogged* stations transmit packets in a slot and $Q_r(i, n)$ the probability that i *backlogged* stations transmit when there are n backlogged stations, we have :

$$Q_a(i, n) = \binom{N-n}{i}(1-q_a)^{N-n-i}q_a^i \quad (9)$$

$$Q_r(i, n) = \binom{n}{i}(1-q_r)^{n-i}q_r^i \quad (10)$$

Slotted Aloha : Stability Analysis III

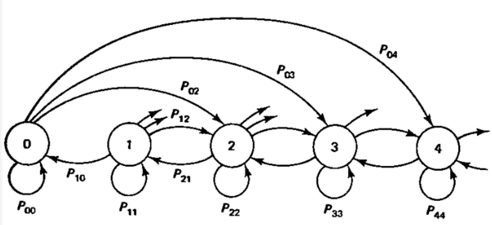
- The transition probabilities can now be written :

$$P(n, n+i) = \begin{cases} Q_a(i,n) & 2 \leq i \leq N-n \\ Q_a(1,n)[1 - Q_r(0,n)] & i = 1 \\ Q_a(1,n)Q_r(0,n) + Q_a(0,n)[1 - Q_r(1,n)] & i = 0 \\ Q_a(0,n)Q_r(1,n) & i = -1 \end{cases} \quad (11)$$

(11a) i unbacklogged stations transmit, collide and become backlogged
(11b) 1 unbacklogged station transmits, fails and becomes backlogged (because of the transmission of some backlogged station)
(11c) 1 unbacklogged station transmits, succeeds and stays unbacklogged OR (no unbacklogged transmits AND there is no successful transmission of a backlogged station)
(11d) 1 backlogged station transmits, succeeds and becomes unbacklogged

Slotted Aloha : Stability Analysis IV

- From which we can deduce the stationary probabilities $\{\pi_n\}_{n=1,\dots,N}$



Slotted Aloha : Stability Analysis V

- The attempt rate in state n is given by :

$$G(n) = (N - n)q_a + nq_r \tag{12}$$

- The probability of successful transmission when there are n backlogged stations is given by :

$$P_{succ}(n) = Q_a(1, n)Q_r(0, n) + Q_a(0, n)Q_r(1, n) \tag{13}$$

If q_a and q_r are small, P_{succ} can be approximated by (using $(1 - x)^y \approx e^{-xy}$) :

$$P_{succ}(n) \approx G(n)e^{-G(n)} \tag{14}$$

- The throughput is then :

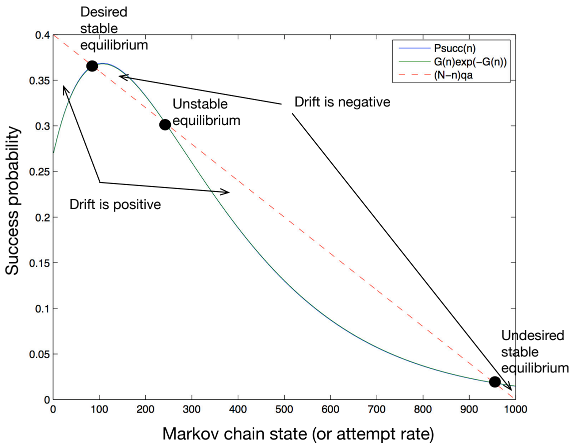
$$S = \sum_{n=1}^{N} P_{succ}(n)\pi_n \tag{15}$$

- The *drift* in state n is defined as the expected change in backlog after one slot, i.e., the number of new arrivals minus the number of successful transmissions :

$$D(n) = (N - n) * q_a - P_{succ}(n) \tag{16}$$

Slotted Aloha : Stability Analysis VI

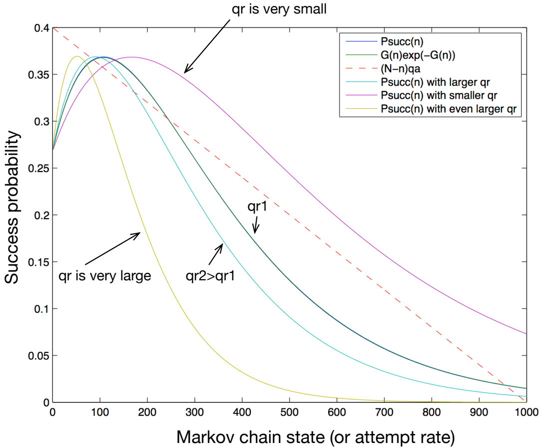
$[q_r > q_a]$



Slotted Aloha : Stability Analysis VII

- Note that for large N the maximum throughput is $1/e$
- The approximation is very good
- The system tends to move in the direction of the drift and to reach equilibrium points (with some excursions between the two)
- When the system is in the undesirable equilibrium state, throughput is close to zero for a long period of time until the desirable equilibrium state is reached again

Slotted Aloha : Stability Analysis VIII



Slotted Aloha : Stability Analysis IX

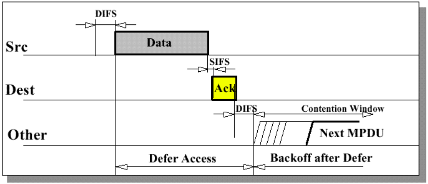
- If q_r is increased (i.e. the delay for retransmissions decreases and the attempt rate increases), the unstable equilibrium gets closer to the desired stable equilibrium and is thus easily reached, so the probability of reaching the undesirable point from the desirable point is higher
- If q_r is further increased, the drift is always positive and the throughput tends to 0
- If q_r is decreased enough, a single equilibrium point remains with a reasonable throughput
- Assuming infinite population, $(N - n)q_a$ (fresh arrivals) becomes λ and the attempt rate becomes $G(n) = \lambda + nq_r$, the straight line becomes horizontal and the undesirable equilibrium disappears : if the system goes beyond the unstable equilibrium, the number of backlogged stations increases indefinitely

CSMA/CA : Basic access I

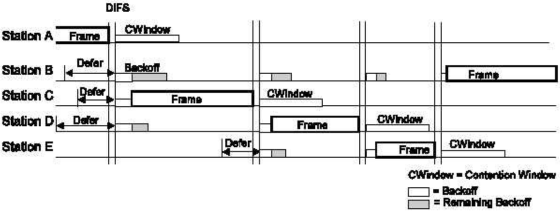
How does it work ?

- Time is slotted but slots (or mini-slots) are much smaller than the packet transmission time as in Aloha.
- Stations have the capability to listen OR transmit (main difference with CSMA/CD).
- Before every transmission, a station launches a timer randomly (uniform) chosen in the interval $[0; CW - 1]$. CW is the *contention window*.
- If the channel is idle during DIFS, the timer is decreased at each idle mini-slot and is blocked when the channel is busy.
- When the timer reaches 0, the station transmits.
- In case of collision, the contention window is doubled $CW = 2^i W$. After m collisions, the window remains constant $CW := CW_{max} = 2^m W$. In case of success, the contention window is set to its minimum value $CW := CW_{min} = 2^0 W$.
- If the Data packet is correctly received, the receiver sends a ACK after SIFS.
- If the Data packet is not correctly received, there is no ACK, and the packet has to be retransmitted.

CSMA/CA : Basic access II



Example :

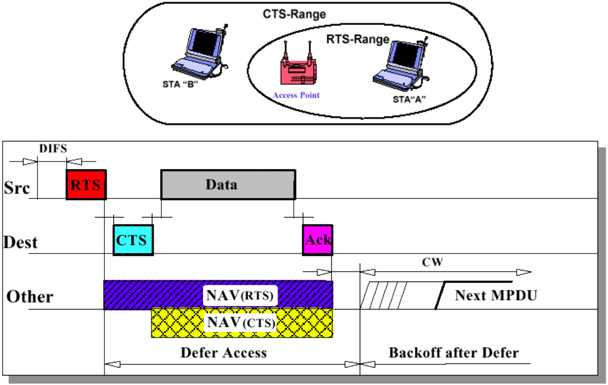


CSMA/CA : RTS and CTS I

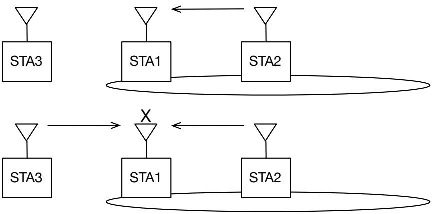
How does it work ?

- The basic access is used to transmit short control messages called RTS (Ready To Send).
- Upon correct reception of a RTS, the receiver transmits a CTS (Clear To Send) after SIFS.
- Upon reception of a CTS, the sender transmits its Data packet after SIFS, which is acknowledged after SIFS.
- Both RTS and CTS carries in their header a NAV (Network Allocation Vector), which specifies how long the Data+ACK transfer will take.
- Any station overhearing the NAV sets as busy the channel until the end of the Data+ACK transfer.

CSMA/CA : RTS and CTS II

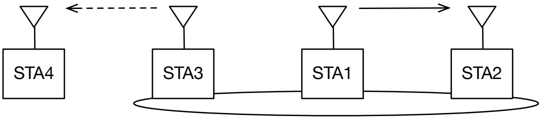


CSMA/CA : Hidden terminal problem



- STA2 is transmitting to STA1
- STA3 is out of sensing range from STA2 and senses the channel idle
- STA3 transmits to STA1 and collides with STA2

CSMA/CA : Exposed terminal problem



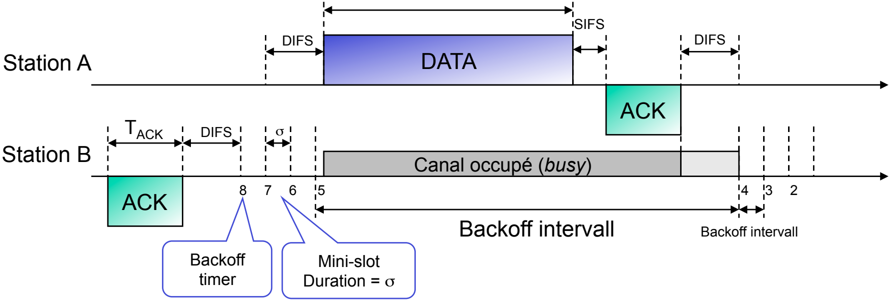
- STA1 is transmitting to STA2
- STA4 is out of range of STA1
- STA3 is in the sensing range of STA1 and prevents itself from transmitting to STA4

CSMA/CA : Analysis I

Assumptions [1]

- N stations, all are in direct visibility (no hidden or exposed terminal)
- All stations have a full buffer (system is saturated)
- Data packets have a constant length and duration T
- Backoff timer is decreased at every start of a mini-slot or after a idle DIFS period
- Backoff intervall : time during which the backoff is constant (may be of variable duration)
- Mini-slot size is σ : this is the required duration for a station to know if the channel is busy or not (duration is constant)
- Let τ be the transmission probability of a station

CSMA/CA : Analysis II



CSMA/CA : Analysis III

Goodput

- There three possible states for a backoff interval :
  1) The interval is idle : its duration is σ, it happens w.p. $(1-\tau)^n$
  2) The interval is busy and the transmission is successful :
  its duration is T_s w.p. $n\tau(1-\tau)^{n-1}$
  3) The interval is busy and there is a collision : duration is T_c w.p.
  $1-(1-\tau)^n - n\tau(1-\tau)^{n-1}$
- Basic access : $T_s = T + SIFS + T_{ACK} + DIFS$ and $T_c = T + DIFS$.
- RTS/CTS access : $T_s = T_{RTS} + 3SIFS + T_{CTS} + T_{ACK} + DIFS + T$ and $T_c = T_{RTS} + DIFS$
- The normalized throughput can thus be written :

$$S = \frac{n\tau(1-\tau)^{n-1}T}{(1-\tau)^n\sigma + n\tau(1-\tau)^{n-1}T_s + \left(1-(1-\tau)^n - n\tau(1-\tau)^{n-1}\right)T_c} \tag{17}$$

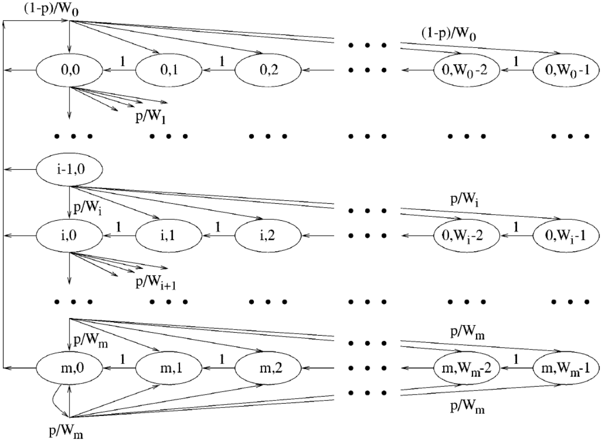
- What about τ ?

CSMA/CA : Analysis IV

Transmission probability τ

- Let's focus on a particular station and let model it as a discrete-time Markov chain
- A state is defined as (s, b), where s is the backoff stage (between 0 and m) and b is the backoff timer value for our station
- State transitions happen at every change of the backoff timer
- Assumption : The probability of collision p for this station is constant and independent on the history

CSMA/CA : Analysis V



CSMA/CA : Analysis VI

- We solve the Markov chain and find the stationary probabilities $P(i,j)$.
- We deduce from $\tau = \sum_{i=0}^{m} P(i,0)$:

$$\tau = \frac{2(1-2p)}{(1-2p)(W+1)+pW(1-(2p)^m)} \tag{18}$$

- We have also :

$$p = 1-(1-\tau)^{n-1} \tag{19}$$

- There is existence and unicity of the solution to this system of two equations and two unknowns. Can be solved numerically.

CSMA/CA : Analysis VII

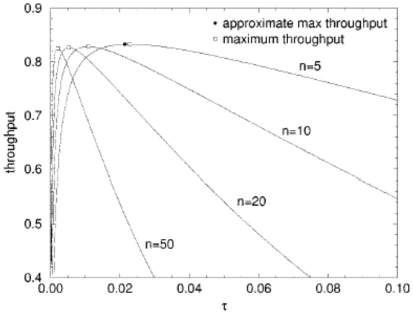


Fig. 7. Throughput versus the transmission probability τ for the basic access method.

[Bianchi00]

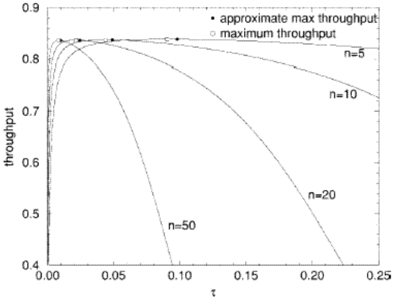


Fig. 8. Throughput versus the transmission probability τ for the RTS/CTS mechanism.

Acronyms

ACK	Acknowledgement
BO	Backoff
CSMA/CA	Carrier Sense Multiple Acces/Collision Avoidance
CSMA/CD	Carrier Sense Multiple Acces/Collision Detection
CTS	Clear To Send
CW	Contention Window
DIFS	Distributed Coordinated Function Interframe Space
MPDU	Medium Access Controm Packet Data Unit
NAV	Network Allocation Vector
PHY	Physical layer
RTS	Ready To Send
SIFS	Short Interframe Space
STA	Station

References

[1] Bianchi, G. (2000). Performance analysis of the IEEE 802.11 distributed coordination function. IEEE Journal on selected areas in communications, 18(3), 535-547.

[2] Bertsekas, D. P., Gallager, R. G., & Humblet, P. (1992). Data networks (Vol. 2). New Jersey : Prentice-Hall International.