Wireless Communications

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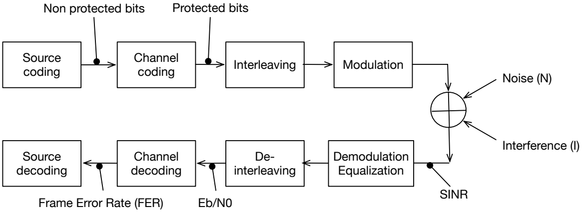
Outlines

Outline

A simplified TRX chain



At the transmitter :

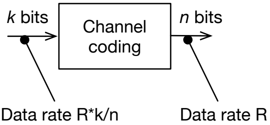
- Source coding : translates the analog signal into a sequence of bits. Alternatively, the source is itself digital (web page, file).
- Channel coding or Forward Error Correction (FEC) : introduces redundancy in the information bit sequence to allow error detection and correction at the receiver.
- Interleaving : coded bits are interleaved in order to mitigate the effect of error bursts.
- Modulation : the process of adapting the coded bits to the medium for the transmission.

A simplified TRX chain

At the receiver :

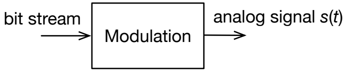
- Demodulation : the process of retrieving the coded bits from the analog signal.
- Equalization : the process of reducing inter-symbol interference (ISI).
- De-interleaving : coded bits are reordered.
- Channel decoding : bit errors are corrected (if possible).
- Source decoding (possibly) : digital to analog signal.

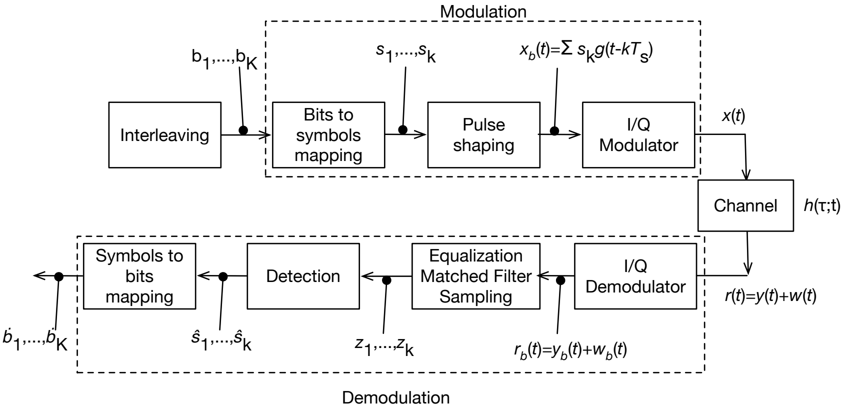
Channel coding



- FEC encoder adds redundant information to the useful bits and the receiver exploits this redundancy to retrieve the original message. FEC is performed at physical layer.
- Code rate is $r = \frac{k}{n}$
- When $r \rightarrow 1$: few redundancy bits, less error correction, high information rate. When $r \rightarrow 0$: many redundancy bits, more error correction, low information rate.
- Examples of codes : linear block codes, convolutional codes, turbo codes, low density parity codes (LDPC), polar codes.

Modulations





Modulations

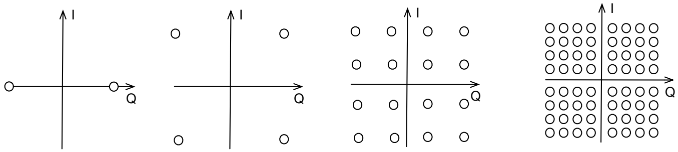
- Bits are grouped into messages of K bits and K-bit messages are sent using symbols every T_s seconds. There are 2^K such symbols.
- Every symbol is mapped to a complex number, called a **signal constellation point** and the set of all these points is called the **signal constellation**.
  Let $\mathcal{C} = \{s^{(0)}, ..., s^{(C-1)}\}$ be the constellation and $C = 2^K$ the number of symbols.
- For a linear modulation, a symbol $s_k \in \mathcal{C}$ is sent during the interval $[kT_s; (k+1)T_s)$ using an analog signal $s_k g(t - kT_s)$. The sequence of transmitted symbols has thus the form :

$$x_b(t) = \sum_k s_k g(t - kT_s) \tag{1}$$

- The data rate is $R_b = \frac{\log_2(C)}{T_s} = \frac{K}{T_s}$.
- x_b is called the complex envelope of the signal and g is called the pulse shape.

Modulations

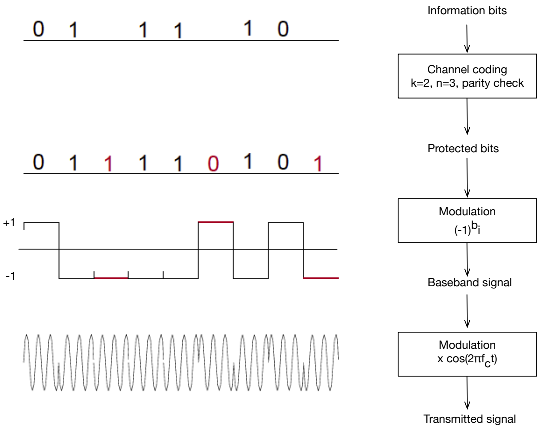
- Examples of linear modulations are : PAM (Pulse Amplitude Modulation), PSK (Phase Shift Keying), QAM (Quadrature Amplitude Modulation)
- Traditional modulations used in wireless networks : BPSK, QPSK (3G), GMSK (GSM), 16-QAM (HSPA), 64-QAM (4G), 256-QAM (5G).
- The denser is the modulation, the more sensitive it is to noise and the higher the number of bits per symbol.



BPSK (1 bit/symbol) QPSK (2 bits/symbol) 16-QAM (4 bits/symbol) 64-QAM (6 bits/symbol)

Modulations

Example : BPSK



Outline

Free space, fixed antennas

- Assume the transmission of a sinusoid $\cos(2\pi ft)$, then the electric far field at time t can be written at $\mathbf{u} = (r, \theta, \phi)$:

$$E_t(f, t, \mathbf{u}) = \frac{\alpha_t(\theta, \phi)}{r} \cos(2\pi f(t - r/c)) \quad (2)$$

  where α_t is the radiation pattern of the sending antenna at f in the direction (θ, ϕ) and c is the speed of light.
- The phase fr/c depends on the delay for the wave to travel to $\mathbf{u}$.
- The electric field amplitude decreases in r^{-1} and the power per square meter decreases in r^{-2}.
- Assume a receive antenna at $\mathbf{u}$, the received electric far field is :

$$E_r(f, t, \mathbf{u}) = \frac{\alpha(\theta, \phi)}{r} \cos(2\pi f(t - r/c)) \quad (3)$$

  where α captures the combined effect of antenna patterns at transmit and receiver sides.

Free space, fixed antennas

- We can define the linear-time invariant channel (linear in the input) as :

$$H(f) \triangleq \frac{\alpha(\theta, \phi) e^{-2j\pi f r / c}}{r} \quad (4)$$

Its inverse Fourier transform is the channel impulse response.
We can write now : $E_r(f, t, \mathbf{u}) = \Re[H(f) e^{2j\pi f t}]$

Free space, moving antennas

- Assume that the receive antenna is moving so that $r(t) = r_0 + vt$.
- Then the received electric field is :

$$E_r(f, t, \mathbf{u}(t)) = \frac{\alpha(\theta, \phi)}{r_0 + vt} \cos(2\pi f(t - r_0/c - vt/c)) \tag{5}$$

$$= \frac{\alpha(\theta, \phi)}{r_0 + vt} \cos(2\pi f(1 - v/c)t - 2\pi f r_0/c) \tag{6}$$

- The Doppler shift is $-fv/c$.

Reflecting wall

- The wave is reflected on a wall at a distance d (with a sign change due to reflection).
- The received signal is a superposition of two waves :

$$E_r(f,t,r) = \frac{\alpha}{r}\cos(2\pi f(t-r/c)) - \frac{\alpha}{2d-r}\cos(2\pi f(t-(2d-r)/c)) \quad (7)$$

- The phase difference is

$$\Delta\theta = \left(\frac{2\pi f(2d-r)}{c} + \pi\right) - \left(\frac{2\pi fr}{c}\right) = \frac{4\pi f}{c}(d-r) + \pi \quad (8)$$

When it is a multiple of 2π, the two waves add constructively. When it is an odd integer multiple of π, they add destructively.

Reflecting wall

- See $\Delta\theta$ as a function of r. Then the distance between a strong signal and a weak signal is
$$\Delta x = \lambda/4 \tag{9}$$
i.e., every Δx, the phase change by π.
$\Delta x = \lambda/4$ and is called the *coherence distance*.
- See $\Delta\theta$ as a function of f. We move also from a maximum to a minimum amplitude if f changes by :
$$\frac{1}{2}\left(\frac{2d-r}{c} - \frac{r}{c}\right)^{-1} \tag{10}$$
The difference between the propagation delays $T_d = \frac{2d-r}{c} - \frac{r}{c}$ is called the *delay spread*. $1/T_d$ is called the *coherence bandwidth*.

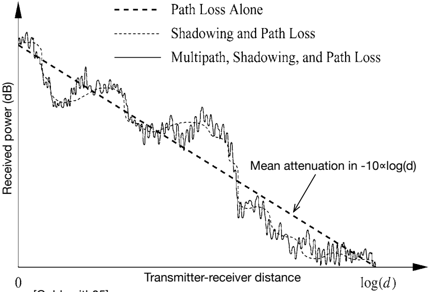
Reflecting wall and moving antenna

- Assume that the receive antenna is moving at velocity v.
- As r is changing, the signal strength is fluctuating between strong and weak. This effect is called *multipath fading* .
- The duration to go from a max to a min is $\Delta x/v = c/(4fv)$ and is called the *coherence time* of the channel.

⇒ The signal strength does no change significantly at a distance less than Δx (coherence distance), at a frequency less than $1/T_d$ (coherence bandwidth) or within a delay less than the coherence time.

The three-stage model

- We characterize the variations of the received power by a 3-stage model.
- Path-loss : dissipation of the radiated power due to distance.
- Shadowing : caused by obstacles on the transmitter-receiver path.
- Fast fading or small-scale fading or multipath fading : variations due to the constructive and destructive addition of multipath signal components.



[Goldsmith05]

Path-loss models

- We start by characterizing the average received power p_r.
- The *linear path-loss and path-gain* are given by :

$$p_\ell = \frac{p_t}{p_r} \tag{11}$$

$$g = \frac{1}{p_\ell} \tag{12}$$

- In dB and dBm, we have :

$$P_t[\text{dBm}] = 10\log_{10}(p_t[\text{mW}]) \tag{13}$$

$$P_r[\text{dBm}] = 10\log_{10}(p_r[\text{mW}]) \tag{14}$$

$$P_L[\text{dB}] = P_t - P_r \tag{15}$$

Path-loss models

- Free-space propagation : there is no obstacle and the signal propagates along a straight line between transmitter and receiver. The free-space channel introduces a simple attenuation and phase offset :

$$y(t) = \Re\{\frac{\lambda\sqrt{g_t g_r}e^{-2\pi jd/\lambda}}{4\pi d}x(t)e^{2\pi jf_c t}\} \tag{16}$$

where g_t and g_r are the transmit and receive antenna gains, λ is the wave length and d is the distance traveled by the wave.

- The free-space path-loss is now given by :

$$p_\ell = \left(\frac{4\pi d}{\lambda\sqrt{g_t g_r}}\right)^2 \tag{17}$$

$$P_L = 10\log_{10}\left(\frac{4\pi}{c\sqrt{g_t g_r}}\right)^2 + 20\log_{10} f_c + 2\times 10\log_{10}(d) \tag{18}$$

- Note : P_L is an increasing function of f_c and d

Path-loss models

- Empirical path-loss models : they are used to model complex environments and are mainly based on extensive measurements campaigns.
- They are valid for a certain range of carrier frequencies and distances and depend on the environment : urban, suburban, rural, indoor, etc.
- Popular models (used for performance evaluation) are : Hata-Okumura, COST231-Hata, COST231-Walfish-Ikegami.
- A simplified path-loss model can be written :

$$p_r(d) = p_t K \left(\frac{d_0}{d} \right)^{\alpha} \tag{19}$$

  where K is a constant that depends on f_c and the environment, $\alpha \geq 2$ is called the *path-loss exponent*, d_0 is a reference distance.
- The path-loss exponent is typically between 2 (free space) and 4 (dense urban).

Path-loss models

- Example : Okumura-Hata for 150-1500 MHz :

$$P_L = A + B\log_{10} d - C \tag{20}$$

with :

$$A = 69.55 + 26.16\log_{10} f_c - 13.82\log_{10} h_b \tag{21}$$

$$B = 44.9 - 6.55\log_{10} h_b \tag{22}$$

$$C = \begin{cases} 3.2(\log_{10}(11.75 f_c))^2 - 4.97 & (Urban) \\ 2(\log_{10}(f_c/28))^2 + 5.4 & (Suburban) \\ 4.78(\log_{10}(f_c))^2 - 18.33\log_{10} f_c + 40.94 & (Rural) \end{cases} \tag{23}$$

(24)

where f_c is in MHz, d in km and h_b (BS antenna height) in m.

Shadowing

- Shadowing : due to obstacles, there are random variations of the received power at a given distance around the average predicted by path-loss models.
- The classical model for shadowing is the log-normal shadowing :

$$p_{a_s}(x) = \frac{10/\ln 10}{\sqrt{2\pi}\sigma x} \exp\left[-\frac{(10\log_{10} x)^2}{2\sigma^2}\right], \quad x > 0 \tag{25}$$

  In dB, shadowing is modeled as a zero-mean Gaussian variable with standard deviation σ :

$$p_{A_s}(X) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{X^2}{2\sigma^2}\right] \tag{26}$$

- Typical standard deviations are from 2 (rural) to 8 dB (urban).
- Assuming distance dependent path-loss and shadowing, we have at d :
  $p_r(d) = \frac{p_t}{p_\ell(d)} a_s$ (in linear, e.g. mW) or $P_r(d) = P_t - P_L(d) + A_s$ (in dBm).

Fast fading : a linear time-varying system

- In general, the received signal is the superposition of several waves aggregating at the receiver along different paths (due to direct propagation, reflections, scattering, diffractions).
- For an input signal $x(t)$, the output of the channel can be written :

$$y(t) = \sum_i a_i(t)x(t-\tau_i(t)) \tag{27}$$

  where a_i and τ_i are the attenuation (due to antenna patterns, reflector nature and location) and the propagation delay at time t along path i (independent on f in narrow-band systems).
- We can write :

$$y(t) = \int_{-\infty}^{+\infty} h(\tau,t)x(t-\tau)d\tau = h(\cdot,t)\star x(t) \tag{28}$$

$$h(\tau,t) \triangleq \sum_i a_i(t)\delta(\tau-\tau_i(t)) \tag{29}$$

Fast fading : a linear time-varying system

- $h(\tau, t)$ is the impulse response of a linear time-varying channel filter. It is the response at time t to an impulse transmitted at time $t - \tau$.
- $H(f; t)$ is the time-varying frequency response (the Fourier Transform of the impulse response) :

$$H(f;t) = \int_{-\infty}^{+\infty} h(\tau, t) e^{-2\pi j f \tau} d\tau \quad (30)$$

$$= \sum_i a_i(t) e^{-2j\pi f \tau_i(t)} \quad (31)$$

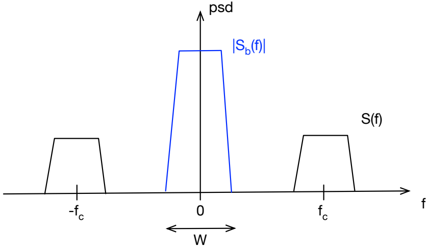
Complex baseband signal

- A typical wireless signal is a passband around a frequency f_c. All signal processing steps are however performed at the baseband before up conversion.
- Consider a real band-limited signal $s(t)$ with Fourier transform $S(f)$ (passband signal). Its complex baseband equivalent (with same energy) is defined as :

$$s(t) = \sqrt{2}\Re[s_b(t)e^{2j\pi f_c t}] \quad (32)$$

$$= s_I(t)\sqrt{2}\cos(2\pi f_c t) - s_Q(t)\sqrt{2}\sin(2\pi f_c t) \quad (33)$$

where $s_I = \Re[s_b]$ and $s_Q = \Im[s_b]$.



- s_I is called the in-phase component and s_Q the quadrature component.

From baseband to passband

I/Q modulation is the process of conversion of the baseband signal $s_b(t)$ to the passband signal (I/Q demodulation for the reverse process).

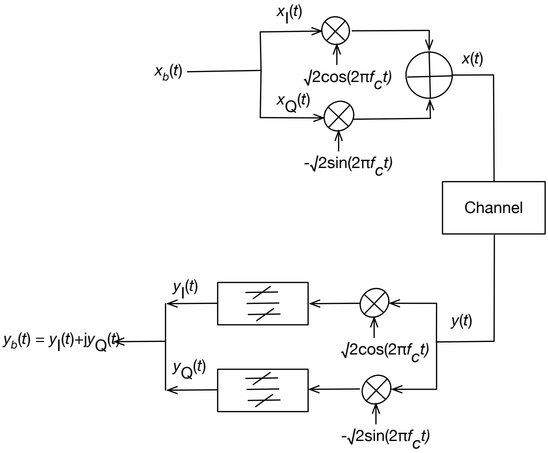
- I/Q modulation is obtained by multiplying two baseband signals by a cos and a sin and by summing the results.
- I/Q demodulation is a multiplication by a cos and a sin and the use of a passband filter :

$$s_b(t)\sqrt{2}\cos(2\pi f_c t) = s_I + s_I \cos(4\pi f_c t) - s_Q \sin(4\pi f_c t) \quad (34)$$

  The first term can be retrieved by filtering the signal around 0 (base band filter).
- As the I/Q modulation and demodulation is straightforward, it is often ignored in simplified transmit/receive chains (TRX) representations.

From baseband to passband

I/Q modulator and demodulator :



Baseband equivalent model

- From $y(t) = h(\cdot, t) \star x(t)$, we can derive the baseband equivalent channel :

$$y_b(t) \quad = \quad \sum_i a_i^b(t) x_b(t - \tau_i(t)) \tag{35}$$

where $a_i^b(t) \triangleq a_i(t) e^{-j2\pi f_c \tau_i(t)}$

- The baseband equivalent impulse response is now :

$$h_b(\tau, t) \quad = \quad \sum_i a_i^b(t) \delta(t - \tau_i(t)) \tag{36}$$

with Fourier Transform :

$$H_b(f; t) \quad = \quad \int_{-\infty}^{+\infty} h(\tau, t) e^{-j2\pi f_c \tau_i(t)} e^{-2\pi j f \tau} d\tau \tag{37}$$

- The baseband output is thus the sum delayed and attenuated replicas of the baseband input.

Discrete time baseband model

- Sampling theorem : any waveform of bandwidth $W/2$ can be expanded in terms of orthogonal basis $\{\text{sinc}(Wt-n)\}_n$ with coefficients given by the samples at integer multiples $1/W$.
- If the input signal $x(t)$ has a band of W, the baseband equivalent has a band of $W/2$ and can be written :

$$x_b(t) = \sum_n x_n \,\text{sinc}(Wt-n) \tag{38}$$

where $x_n = x_b(n/W)$ and $\text{sinc}(x) = \sin(\pi x)/(\pi x)$.

Discrete time baseband model

- The baseband output is now :

$$y_b(t) = \sum_n x_n \sum_i a_i^b(t)\,\mathrm{sinc}(Wt - W\tau_i(t) - n) \quad (39)$$

and when sampled at multiples of $1/W$ ($y_m = y_b(m/W)$) :

$$y_m = \sum_n x_n \sum_i a_i^b(m/W)\,\mathrm{sinc}(m - n - \tau_i(m/W)W) \quad (40)$$

$$= \sum_\ell x_{m-\ell} \sum_i a_i^b(m/W)\,\mathrm{sinc}(\ell - \tau_i(m/W)W) \quad (41)$$

Discrete time baseband model

- We have now a discrete time baseband model

$$y_m = \sum_{\ell} h_{\ell,m} x_{m-\ell} \quad (42)$$

  where $h_{\ell,m}$ is the ℓ-th complex channel filter tap at time m.
- The complex value x_n is the n-th sample of the input signal.
- For a time-invariant channel, i.e. a_i^b and τ_i are independent on time, $h_{\ell,m} = h_\ell$ is independent on m and we have :

$$y_m = \sum_{\ell} h_\ell x_{m-\ell} \quad (43)$$

Discrete time baseband model

Additive zero-mean White Gaussian Noise with power spectral density $N_0/2$.

- The discrete time baseband model is now :

$$r_m = \sum_{\ell} h_{\ell,m} x_{m-\ell} + w_m \tag{44}$$

- where w_n is white (independent over time), has Gaussian real and imaginary parts with variances $N_0/2$ that are independent (i.e. *circular symmetric complex Gaussian*).

Rayleigh fading

- The ℓ-th filter tap of the channel can be written :

$$h_{\ell,m} = \sum_i a_i^b(m/W)e^{-2j\pi f_c \tau_i(m/W)} \operatorname{sinc}(\ell - \tau_i(m/W)W) \tag{45}$$

- This tap is the sum of a large number of independent complex random variables with uniform phase.
- Using the Central Limit Theorem, $h_{\ell,m}$ can be approximated by a circularly symmetric Gaussian random variable $\mathcal{CN}(0, \sigma_\ell^2)$.
- Its amplitude $|h_{\ell,m}|$ is a Rayleigh random variable with pdf :

$$\frac{x}{\sigma_\ell^2} \exp\left(-\frac{x^2}{2\sigma_\ell^2}\right) \tag{46}$$

Summary (in absence of noise)

- Real signals :

$$y(t) = \sqrt{2}\Re\left\{\left[\sum_i a_i(t)e^{-2j\pi f_c\tau_i(t)}x(t-\tau_i(t))\right]e^{2j\pi f_c t}\right\} \quad (47)$$

$$h(\tau;t) = \sum_i a_i(t)e^{-2j\pi f_c\tau_i(t)}\delta(t-\tau_i(t)) \quad (48)$$

- Equivalent complex baseband signals :

$$y_b(t) = \int_{-\infty}^{+\infty} h_b(\tau,t)x_b(t-\tau)d\tau \quad (49)$$

$$= \int_{-\infty}^{+\infty} H_b(f;t)X_b(f)e^{2j\pi ft}df \quad (50)$$

$$h_b(\tau;t) = \sum_i a_i^b(t)\delta(\tau-\tau_i(t)) \quad (51)$$

- Discrete baseband model :

$$y_m = \sum_\ell h_{\ell,m}x_{m-\ell} \quad (52)$$

Important channel characteristics

- Delay spread : $\tau_{\max} = \max_i \tau_i - \tau_1$ this is the dispersion of the signal in time.
- Coherence bandwidth : this is the maximal variation W_c such that $H_b(f, t) \approx H_b(f + W_c; t)$, i.e, the channel is approximately constant over a bandwidth W_c. We have :
$$W_c \approx \frac{1}{\tau_{\max}}$$
- Coherence time : this is the maximal variation T_c such that $h_b(\tau; t) \approx h_b(\tau; t + T_c)$, i.e., the channel is approximately constant over an interval of time less than T_c.
- Maximum Doppler shift : maximum deviation of an impulse in the frequency domain through the channel. $W_d = \frac{vf_c}{c}$, where v is the terminal speed and c the speed of light. We have :
$$W_d \approx \frac{1}{T_c}$$

Some typical channels

- **Gaussian channel (AWGN)** : $\tau_{\max} << T_s$ and $T_s << T_c$.
  Time dispersion is very low compared to the symbol duration and the channel remains constant over a long period of time (time-invariant). Assuming wlog $a_1^b = 1$:

  Baseband model : $r_b(t) = x_b(t) + w_b(t)$.

  Discrete model : $r_m = x_m + w_m$.

Some typical channels

- **Flat fading channel** : $W << W_c$ (or eq. $T_s >> \tau_{\max}$) and $T_s \approx T_c$.
  The channel is almost constant over the signal bandwidth but varies at every symbol duration.

  We have $x_b(t - \tau_i) \approx x_b(t)$, so that :

$$r_b(t) = x_b(t) \sum_i a_i^b(t) + w_b(t) \quad (53)$$

$$r_m = h_m x_m + w_m \quad (54)$$

  Assuming uniform distribution of the phases (no line-of-sight propagation), $h_m \in \mathcal{CN}(0, 1)$ and the envelope $|h_m|$ is Rayleigh distributed.

  The distribution of the received power is exponential :

$$\frac{1}{p_r} e^{-\frac{z}{p_r}} \quad (55)$$

  where p_r is the average received signal power (obtained from path-loss and shadowing).

Some typical channels

- **Frequency selective channel** : $\tau_{\max} > T_s$ (or eq. $W > W_c$) and $T_s << T_c$.
  The channel is almost time-invariant but affects the signal differently according to the frequency :

$$r_b(t) = \int H_b(f; t_0) X_b(f) e^{2j\pi f t_0} df + w_b(t) \tag{56}$$

  Multiple copies of the same delayed signal are received at the receiver, this is the Intersymbol Interference (ISI).

Outline

Demodulation

- Consider the following receive complex signal :

$$r_b(t) = \sum_k s_k c(t - kT_s) + w_b(t) \tag{57}$$

where $c(t) = h_b(\cdot; t) \star g(t)$ is the equivalent channel.

- In AWGN or flat fading channels, the receive filter that maximizes the SNR at the output of the demodulator is the **matched filter** : $z(t) = c^*(-t) \star r_b(t)$.
- Demodulation consists in filtering and sampling the signal as follows :

$$z(t) = \sum_k s_k c^*(-t) \star c(t - kT_s) + c^*(-t) \star w(t) \tag{58}$$

$$z_n = \sum_k s_k c'_{n-k} + w'_n \tag{59}$$

$$= s_n c'_0 + \sum_{k \neq n} s_k c'_{n-k} + w'_n \tag{60}$$

where $z_n = z(nT_s)$, $c'_n = c'(nT_s)$, $c'(t) = c^*(-t) \star c(t)$, $w'_n = w'(nT_s)$, $c'(t) = c^*(-t) \star w(t)$.

Demodulation

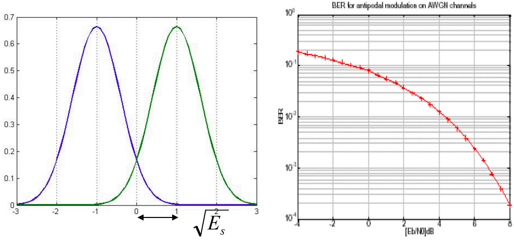
- If the filter has the property that $c'_n = 0$ if $n \neq 0$ and $c'_0 = 1$, we say that this filter has the Nyquist property. A necessary condition is $1/T_s > 2W$.
- In AWGN or flat fading, one can design a pulse shape $g(t)$ and the corresponding matched filter $g^*(-t)$ so that the Nyquist property is verified (root raised cosine is an example). There is no inter-symbol interference.
- In frequency selective channels, the term $\sum_{k \neq n} s_k c'_{n-k}$ cannot be cancelled and ISI appears.
- Techniques to combat ISI are : equalization, multicarrier transmission, and spread spectrum.

Detection

- In BPSK, there is one bit per symbol $s \in \{-1, +1\}$.
- Consider the event A_{01} : -1 is sent but the received signal $r = \sqrt{E_s}s + w$ is closer to $+\sqrt{E_s}$.

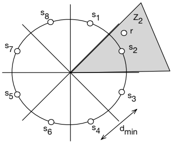
$$\mathbb{P}[A_{01}] = \mathbb{P}[w > \sqrt{E_s}] = Q\left(\sqrt{\frac{2E_s}{N_0}}\right) \tag{61}$$

where E_s is the received signal energy per symbol (i.e. bit here). $N_0/2$ is the variance of AWGN.



Detection

- Imagine we estimate at the receiver from received signal r a symbol $\hat{s}$ while a symbol s_i has been transmitted.
- The Maximum Likelihood (ML) receiver computed $\hat{s} \triangleq \text{argmax}_{s_j} \mathbb{P}[r|s_j \text{ sent}]$. ML is the receiver that minimizes the error probability.
- This is equivalent to finding the s_i that is the closest to r :
  $Z_i = \{x||x - s_i| < |x - s_j| \; \forall j\}$
  $r \in Z_i \Rightarrow \hat{s} = s_i$



- For a constellation with M points, we have :

$$P_e \leq (M-1)Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) \tag{62}$$

Performance parameters

Some radio quality parameters :

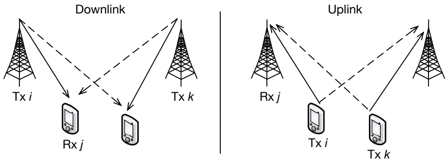
- SINR : A power ratio at the entrance of the reception chain. SINR thresholds are characteristics of the services and of the system.
- E_b/N_0 : An energy ratio (bit energy over power spectral density of noise and interference). Eb/N0 thresholds mainly depend on the considered service, e.g., for voice $E_b/N_0 \approx 6$ dB. Channel coding can be seen as a booster for reducing Eb/N0 threshold.
- FER, BER (Bit Error Rate), BLER (Block Error Rate) : Ratio of correct frame, block or bits after error correction.

Performance parameters : SINR

- The quality of the received radio signal is measured by the Signal to Interference plus Noise Ratio (SINR).
- Interference means here "co-channel interference", i.e., interference is received from transmitters using the same carrier frequency f_c.
- Let consider a transmitter i with transmit power p_i and a receiver j. Let g_{ij} be the channel gain between i and j. We have at j :

$$\gamma_{ij} = \frac{p_i g_{ij}}{\sum_{k \neq i} p_k g_{kj} + N} \tag{63}$$

- N is the thermal (background) noise power. $N = N_0 W$, where N_0 is the noise power spectral density : $N_0[dBm] = 10 \log_{10}(kT)$, where $k = 1.38066 \cdot 10^{-23}$ J/K is the Boltzmann constant and T is the temperature in Kelvin.



Performance parameters : outage probability

- Outage probability in terms of received power :

$$P_{out} = \mathbb{P}[P_r(d) \leq P_{\min}] = 1 - Q\left(\frac{P_{\min} - (P_t - P_L)}{\sigma}\right) \quad (64)$$

where $Q(z) \triangleq p(x > z) = \int_z^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$ and x is a normal r.v. (zero mean, variance one). Note : $Q(z) = \frac{1}{2} erfc(z/\sqrt{2})$.

- Outage probability in terms of SINR :

$$P_{out} = \mathbb{P}[\gamma \leq \gamma_{\min}] \quad (65)$$

Outline

Capacity of wireless channels : AWGN

- Assume a discrete-time additive white Gaussian noise (AWGN) channel $r_m = x_m + w_m$, where x_m is the channel input at time m, r_m is the channel output and w_m is a white Gaussian noise random process.
- Let P be the power of x and W the signal bandwidth. Let $\gamma = \frac{P}{N_0 W}$ the SNR, where N_0 is the power spectral density of noise.
- The capacity of this channel in bits/seconds is :

$$C = W \log_2(1 + \gamma) \tag{66}$$

- Shannon's coding theorem states that there exists a (channel) code that achieves data rates arbitrarily close to C with arbitrarily small probability of error.
- This capacity is often used as an upper bound on the data rates achieved in reality. Today's codes and implementations are very close to Shannon capacity. Turbo codes for ex. are within a fraction of dB of Shannon capacity over AWGN.

Capacity of wireless channels : flat fading

- Assume now the following channel model : $y_m = \sqrt{g_m} x_m + w_m$, where g_m is an i.i.d. random process. Let P be the transmit signal power. The instantaneous SNR is given by $\gamma_m = \frac{P g_m}{N_0 W}$ and is assumed to follow a distribution $p_\gamma(x)$.
- Channel Side Information (CSI) is g_m.
- If CSI is known at the receiver for every m, Shannon capacity is given by :

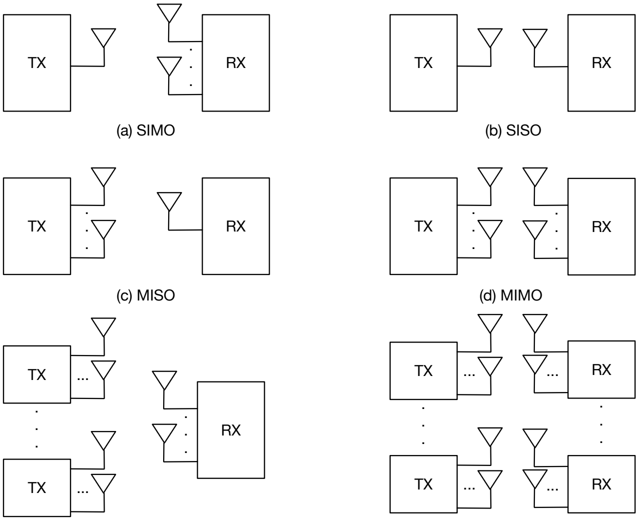
$$C = \mathbb{E}_\gamma[W \log_2(1+\gamma)] = \int_0^\infty W \log_2(1+x) p_\gamma(x) dx \tag{67}$$

Outline

Multiple antenna systems : potential gains

- Multiple antennas are used to increase the system throughput or the robustness of the signal to fading.
- The possible gains with MIMO :
  - **Diversity gain :** several independent copies of the signal are available at the receiver, improves the robustness of the signal (in termes of SNR or BER).
  - **Array gain :** transmission side : energy is focused in one or several directions *(beamforming)*, reception side : more energy is captured by several antennas, improves the SNR, may reduce interferences.
  - **Multiplexing gain :** several parallel flows are simultaneously transmitted, the SISO channel capacity is multiplied by $\min(M, N)$, where N and M are the number of transmit and receive antennas resp.

Multiple antenna systems : typology



(a) SIMO (b) SISO

(c) MISO (d) MIMO

(e) MU-MIMO (f) Network MIMO

Multiple antennas : receive diversity

- Assume one TX antenna and two RX antenna, flat fading channel, and CSI known at receiver.
- The receive signals at the antennas are :

$$y_1 = h_1 s + n_1 \tag{68}$$
$$y_2 = h_2 s + n_2 \tag{69}$$

- At the receiver, we combine the received signals :

$$r = \mathbf{w}^H \mathbf{y} = w_1^* y_1 + w_2^* y_2 = s(w_1^* h_1 + w_2^* h_2) + (w_1^* n_1 + w_2^* n_2) \tag{70}$$

- The coefficients that maximizes SNR are $w_i = h_i$ (Maximum Ratio Combining). We obtain :

$$\gamma = \frac{(|h_1|^2 + |h_2|^2)E_s}{N_0} \tag{71}$$

- If h_1 and h_2 are independent, a diversity order of 2 is achieved (not shown here) in the sense that the SER decreases in $(E_s/N_0)^{-2}$. Array gain is 2.

Multiple antennas : transmit diversity

- Assume two TX antennas and one RX antenna, flat fading channel and CSI unknown at transmitter.
- **Alamouti scheme :** At T_s, s_1 and s_2 are simultaneously transmitted from the two antennas. At $T_s + 1$, $-s_2^*$ is sent from antenna 1 and s_1^* is sent from antenna 2.
- The received vector $\boldsymbol{y} = [y_1; y_2^*]^T$ is :

$$\boldsymbol{y} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n} \tag{72}$$

We have : $\boldsymbol{H}^H \boldsymbol{H} = (|h_1|^2 + |h_2|^2)\boldsymbol{I}_2$

- If the CSI is known at the receiver, it decodes as follows :

$$\boldsymbol{z} \triangleq \boldsymbol{H}^H \boldsymbol{y} = (|h_1|^2 + |h_2|^2)\boldsymbol{I}_2 s + \tilde{\boldsymbol{n}} \tag{73}$$

where $\tilde{\boldsymbol{n}} = \boldsymbol{H}^H \boldsymbol{n}$ has energy $(|h_1|^2 + |h_2|^2)N$. The SNR for z is now :

$$\gamma = \frac{(|h_1|^2 + |h_2|^2)E_s}{2N_0} \tag{74}$$

where the factor 2 comes from the fact that half of the total energy is used on each antenna. Diversity order of 2 is also achieved but array gain is 1.

Multiple antennas : spatial multiplexing I

- Assume a flat fading channel :

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n} \tag{75}$$

where $\boldsymbol{H} = (h_{mn})_{m=1\ldots M,\ n=1\ldots N}$ is the channel matrix, N is the number of transmit antennas, M is the number of receive antennas, $\boldsymbol{y} = [y_1, \ldots, y_M]^T$ is the received signal, $\boldsymbol{s} = [s_1, \ldots, s_N]^T$ is the transmitted signal and $\boldsymbol{n} = [n_1, \ldots, n_M]^T$ is noise, modeled as zero-mean circularly symmetric complex Gaussian with variance σ_n^2 (i.e., $E[\boldsymbol{n}] = 0$, $E[\boldsymbol{n}\boldsymbol{n}^T] = 0$).
On the m-th antenna, the received signal is thus :

$$y_m = \sum_{n=1}^{N} h_{nm} s_n + n_m \tag{76}$$

- The MIMO channel can be decomposed into R parallel independent channels.

Multiple antennas : spatial multiplexing II

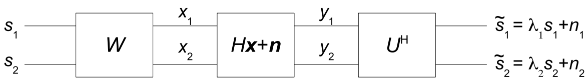
- For a $M \times N$ matrix $\boldsymbol{H}$, the Singular Value Decomposition (SVD) can be written :

$$\boldsymbol{H} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{W}^H \tag{77}$$

where $\boldsymbol{U}$ $(M \times M)$ and $\boldsymbol{W}$ $(N \times N)$ are unitary matrices (in particular $\boldsymbol{U}\boldsymbol{U}^H = I_M$ and $\boldsymbol{W}^H\boldsymbol{W} = I_N$) and $\boldsymbol{\Sigma}$ is a $M \times N$ diagonal matrix of singular values (σ_i) of $\boldsymbol{H}$.

- We have $\sigma_i = \sqrt{\lambda_i}$, where λ_i is the i-th eigenvalue of $\boldsymbol{H}\boldsymbol{H}^H$. There are R_H non-zero eigenvalues if $\boldsymbol{H}$ has rank $R_H \leq \min\{M, N\}$. The channel is said to be full rank if $R_H = \min\{M, N\}$.
- Channel decomposition is obtained by performing transmit precoding and receiver shaping :

$$\begin{aligned} \tilde{\boldsymbol{s}} &= \boldsymbol{U}^H((\boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{W}^H)(\boldsymbol{W}\boldsymbol{s}) + \boldsymbol{n}) && (78) \\ &= \boldsymbol{\Sigma}\boldsymbol{s} + \tilde{\boldsymbol{n}} && (79) \end{aligned}$$



Multiple antennas : MIMO capacity I

- Over a static channel, with channel unknown at the transmitter and uniform power allocation :

$$C = W \log_2 \det(\boldsymbol{I}_M + \frac{\rho}{N} \boldsymbol{H}\boldsymbol{H}^H) = \sum_{i=1}^{R_H} W \log_2(1 + \frac{\sigma_i^2 \rho}{N}) \qquad (80)$$

where $\rho = P/\sigma_n^2$, P is the transmit power, σ_n^2 is the noise power.

- Over a static channel, with channel known at the transmitter, power allocation can be optimized :

$$C = \max_{P_i : \sum_i P_i \leq P} \sum_{i=1}^{R_H} W \log_2(1 + \frac{P_i \sigma_i^2}{\sigma_n^2}) = \max_{P_i : \sum_i P_i \leq P} \sum_{i=1}^{R_H} W \log_2(1 + \frac{P_i \gamma_i}{P}) \qquad (81)$$

where $\gamma_i = \sigma_i^2 P/\sigma_n^2$.

Multiple antennas : MIMO capacity II

Solving the optimization problem leads to a *waterfilling* allocation :

$$\frac{P_i}{P} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma_i} & \text{if} \gamma_i \geq \gamma_0 \\ 0 & \text{otherwise} \end{cases} \quad (82)$$

where the cut-off value γ_0 is such that $\sum_i P_i = P$.

The resulting capacity is :

$$C = \sum_{i:\gamma_i \geq \gamma_0} W \log(\gamma_i/\gamma_0)$$

- In fading channels, the channel is modeled as zero-mean circularly symmetric unit variance matrix. With channel unknown at the transmitter and uniform power allocation, we have :

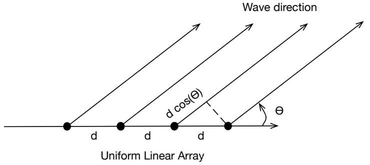
$$C = \mathbb{E}_H[W \log_2 \det(\boldsymbol{I}_M + \frac{\rho}{N}\boldsymbol{H}\boldsymbol{H}^H)] \quad (83)$$

Multiple antennas : beamforming I

- Beamforming is obtained by coherently summing signals from multiple antennas.
- Example : the Uniform Linear Array. Assume N identical omnidirectionnal antennas. Signal on antenna i is multiplied by $w_i = \frac{1}{\sqrt{N}} e^{\frac{-j2\pi id\cos\theta_0}{\lambda}}$ (phase offset), where λ is the wave length. The equivalent antenna pattern is given by :

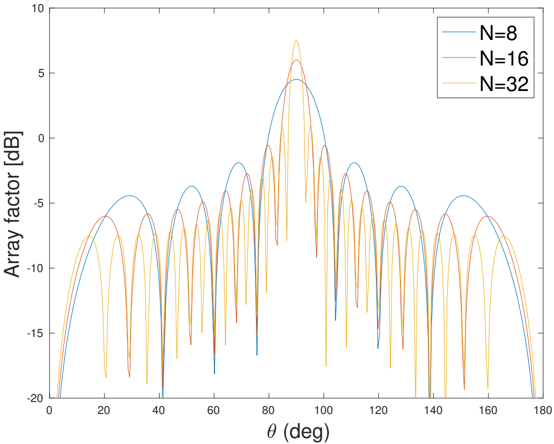
$$g_N(\theta, \phi) = \left| \sum_{i=0}^{N-1} w_i e^{\frac{j2\pi id\cos\theta}{\lambda}} \right| = \frac{1}{\sqrt{N}} \left| \frac{\sin N\gamma/2}{\sin\gamma/2} \right| \quad (84)$$

where $\gamma = \frac{2\pi d(\cos\theta - \cos\theta_0)}{\lambda}$.
g_N is called the array factor. It is maximum for $\theta = \theta_0$ (main lobe). In the main lobe direction, $g_N = \sqrt{N}$ in amplitude, i.e., a factor of N in power.



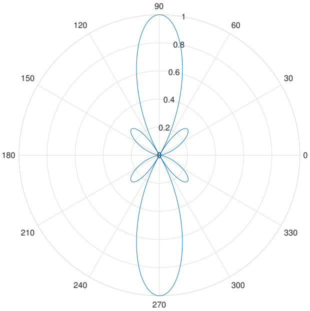
Multiple antennas : beamforming II

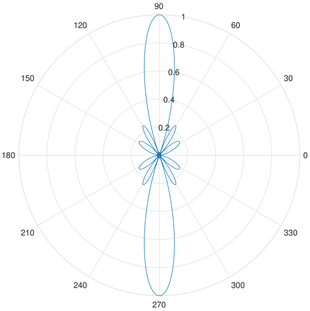
$N = 8,\ 16,\ 32,\ d = \lambda/2,\ \theta_0 = \pi/2$



Multiple antennas : beamforming III

$N = 4$ and 6





Acronyms I

AWGN	Average White Gaussian Noise
BER	Bit Error Rate
BLER	Block Error Rate
BPSK	Binary Phase Shift Keying
BS	Base Station
CDMA	Code Division Multiple Access
CSI	Channel Side Information
CSMA/CA	Carrier Sense Multiple Acces/Collision Avoidance
DL	Downlink
FDD	Frequency Division Duplex
FDMA	Frequency Division Multiple Access
FEC	Forward Error Correction
FER	Frame Error Rate
GMSK	Gaussian Minimum Shift Keying
ISI	Inter-Symbol Interference
LDPC	Low Density Parity Codes
MIMO	Multiple Input Multiple Output
MISO	Multiple Input Single Output
ML	Maximum Likelihood
MS	Mobile Station
MU-MIMO	Multi-User MIMO
OFDM	Orthogonal Frequency Division Multiplex
OFDMA	Orthogonal Frequency Division Multiple Access
OMA	Orthogonal Multiple Access
PAM	Pulse Amplitude Modulation
PDU	Packet Data Unit
PSK	Phase Shift Keying
QAM	Quadrature Amplitude Modulation

Acronyms II

QPSK	Quadrature Phase Shift Keying
RX	Reception
SDMA	Space Division Multiple Access
SER	Symbol Error Rate
SIC	Successive Interference Cancellation
SINR	Signal to Interference plus Noise Ratio
SIMO	Single Input Multiple Output
SISO	Single Input Single Output
SNR	Signal to Noise Ratio
TDD	Time Division Duplex
TDMA	Time Division Multiple Access
TS	Time-Slot
TRX	Transmission and Reception
TX	Transmission
UE	User Equipment
UL	Uplink

Outline

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