



Joint Impact of Pathloss, Shadowing and Fast Fading - An Outage Formula for Wireless Networks

Impact conjoint de l'affaiblissement de parcours, de l'effet de masque et des évanouissements rapides - Une formule de la probabilité de dépassement pour les réseaux sans fil

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Abstract

In this report, we analyse the joint impact of path-loss, shadowing and fast fading on wireless networks. Taking into account the pathloss and the shadowing, we first express the SINR distribution of a mobile located at a given distance from its serving base-station (BS). The moments of this distribution are easily computed, using the Fenton-Wilkinson approximation, and a fluid model that considers the cellular network as a continuum of BS. Then considering the joint impact of pathloss, shadowing and fast fading, we derive an easily computable outage probability formula, for a mobile located at any distance from its serving BS. We validate our approach by comparing all results to Monte Carlo simulations performed in a traditional hexagonal network. Indeed, we establish that the results given by the formula are close to the ones given by Monte Carlo simulations. The proposed framework is a powerful tool to study performances of cellular networks e.g. OFDMA systems (WiMAX, LTE).

Dans ce rapport, nous analysons l'impact conjoint de l'affaiblissement de parcours, de l'effet de masque et des évanouissements rapides sur les réseaux sans fil. Nous exprimons la distribution du SINR d'un mobile situé à une certaine distance de sa station de base (BS) serveuse. Les moments de la distribution sont calculés en utilisant l'approximation de Fenton-Wilkinson et un modèle fluide qui considère le réseau cellulaire comme un continuum de stations de base. Nous pouvons en déduire une formule simple de la probabilité de dépassement pour ce mobile. Nous validons notre approche en comparant les résultats à ceux obtenus avec des simulations de Monte Carlo prenant comme hypothèse un réseau hexagonal. Le cadre proposé représente un outil puissant pour l'analyse des performances des réseaux cellulaires.

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1 Introduction

The characterization of interference allows to analyze cellular networks performance. In cellular networks, an important parameter for this characterization is the SINR (signal to interference plus noise ratio) at any point of the cell. Indeed, it allows the derivation of performance evaluation (outage probabilities, capacity,...). We focus on downlink throughout this report, since that direction is often the limited link in term of capacity. Nevertheless, our approach can easily be extended to the uplink.

The issue of expressing outage probability in cellular networks has been extensively addressed in the literature. For this study, there are two possible assumptions: (1) considering only the shadowing effect, (2) considering both shadowing and fast fading effects. In the former case, authors mainly face the problem of expressing the distribution of the sum of log-normally random variables; several classical methods can be applied to solve this issue (see e.g. [11] [3]). In the latter case, formulas usually consist in many infinite integrals, which are uneasy to handle in a practical way (see e.g. [12]). In both cases, outage probability is always an explicit function of all distances from the user to interferers.

As the need for easy-to-use formulas for outage probability is clear, approximations need to be done. Working on the uplink, [10] derived the distribution function of a ratio of path-losses with shadowing, which is essential for the evaluation of external interference. For that, authors approximate the hexagonal cell with a disk of same area. Authors of [13] assumes perfect power control on the uplink, while neglecting fast fading. On the downlink, Chan and Hanly [8] precisely approximate the distribution of the other-cell interference. They however provide formulas that are difficult to handle in practice and do not consider fast fading. Immovilli and Merani [14] take into account both channel effects and make several assumptions in order to obtain simplified formulas. In particular, they approximate interference by its mean value. Outage probability is however an explicit function of all distances from receiver to every interferer.

In the aim of calculating the SINR distribution, taking into account the shadowing, we approximate a sum of log-normal random variables (RV) by a log-normal RV using the Fenton-Wilkinson method. Then, we assume that the fast fading factor of the useful signal has a dominant impact on the outage probability. That assumption

enables to jointly take into account shadowing and fast fading and to derive an easily computable outage probability formula, for a mobile located at any distance from its serving BS. That assumption is validated through Monte Carlo simulations. At last, we rely on a recently proposed fluid model [5] [6] in order to express the outage probability as a function of the distance to the serving BS. Such an expression allows further integrations much more easily than with existing formulas. The formula can be used to analyze the coverage of a BS.

In the next section, we derive outage probabilities, while considering first only shadowing and then shadowing and fast fading jointly. The computation is based on the fluid model (section 3) and on an assumption on the Rayleigh fading. In section 4, we validate our approximation and compare analytical expressions with results obtained through Monte Carlo simulations.

2 Outage Probabilities

We consider N interfering base station (BS), a mobile u and its serving BS_0 .

2.1 Shadowing impact

2.1.1 Propagation

Considering the power P_j transmitted by the BS j , the power $p_{j,u}$ received by a mobile u can be written:

$$p_{j,u} = P_j K r_{j,u}^{-\eta} Y_{j,u}, \quad (1)$$

where $Y_{j,u} = 10^{\frac{\xi_{j,u}}{10}}$ represents the shadowing effect. The term $Y_{j,u}$ is a lognormal random variable characterizing the random variations of the received power around a mean value. $\xi_{j,u}$ is a *Normal* distributed random variable (RV), with zero mean and standard deviation, σ , comprised between 0 and 10 dB. The term $P_j K r_{j,u}^{-\eta}$, where K is a constant, represents the mean value of the received power at distance $r_{j,u}$ from the transmitter (BS_j). The probability density function (PDF) of this slowly varying received power is given by

$$p_Y(s) = \frac{1}{a\sigma s\sqrt{\pi}} \exp - \left(\frac{\ln(s) - am}{\sqrt{2}a\sigma} \right)^2 \quad (2)$$

where $a = \frac{\ln 10}{10}$, $m = \frac{1}{a} \ln(KP_j r_{j,u}^{-\eta})$ is the (logarithmic) received mean power expressed in decibels (dB), which is related to the path loss and σ is the (logarithmic) standard deviation of the mean received signal due to the shadowing.

2.1.2 SINR calculation

Considering the useful power P_0 transmitted by the BS_0 , the useful power $p_{0,u}$ received by a mobile u belonging to BS_0 can be written:

$$p_{0,u} = P_0 K r_{0,u}^{-\eta} Y_{0,u}. \quad (3)$$

For the sake of simplicity, we now drop index u and set $r_{0,u} = r$. The interferences received by u coming from all the other base stations of the network are expressed by:

$$p_{ext} = \sum_{j=1}^N P_j K \cdot r_j^{-\eta} Y_j. \quad (4)$$

The SINR at user u is given by:

$$\gamma = \frac{P_0 K r^{-\eta} Y_0}{\sum_{j=1}^N P_j K r_j^{-\eta} Y_j + N_{th}}. \quad (5)$$

If now, thermal noise is negligible and BS have identical transmitting powers, the SINR can be written $\gamma = 1/Y_f$ with

$$Y_f = \frac{\sum_{j=1}^N r_j^{-\eta} Y_j}{r^{-\eta} Y_0}. \quad (6)$$

2.1.3 Distribution of Y_f

The factor Y_f is defined for any mobile u and it is location dependent. The numerator of this factor is a sum of log-normally distributed RV, which can be approximated by a log-normally distributed RV [3]. The denominator of the factor is a log-normally distributed RV. Y_f can thus be approximated by a log-normal RV.

Using the Fenton-Wilkinson [1] method, we can calculate the mean and standard deviation, m_f and s_f , of factor Y_f , for any mobile at the distance r from its serving BS, BS_0 . We first calculate the mean and the variance of a sum of log-normal RV

(appendix 1). We afterwards apply the result to the sum of N log-normal identically distributed RV (appendix 2):

$$m_f = \frac{1}{a} \ln(y_f(r, \eta)H(r, \sigma)), \quad (7)$$

$$s_f^2 = 2(\sigma^2 - \frac{1}{a^2} \ln H(r, \sigma)), \quad (8)$$

where

$$H(r, \sigma) = e^{a^2\sigma^2/2} \left(G(r, \eta)(e^{a^2\sigma^2} - 1) + 1 \right)^{-\frac{1}{2}}, \quad (9)$$

$$G(r, \eta) = \frac{\sum_j r_j^{-2\eta}}{\left(\sum_j r_j^{-\eta} \right)^2}, \quad (10)$$

$$y_f(r, \eta) = \frac{\sum_j r_j^{-\eta}}{r^{-\eta}}. \quad (11)$$

From Eq. 6 and 11, we notice that $y_f(r, \eta)$ represents the factor Y_f *without shadowing*.

2.1.4 Outage probability

The outage probability is defined as the probability for the SINR γ to be lower than a threshold value δ and can be expressed as:

$$\mathbb{P}(\gamma < \delta) = 1 - \mathbb{P}(p_0 > \delta(p_{ext} + N_{th})) \quad (12)$$

So we have:

$$\mathbb{P}(\gamma < \delta) = 1 - \mathbb{P}\left(\frac{1}{\delta} > Y_f(m_f, s_f)\right) \quad (13)$$

$$= 1 - \mathbb{P}\left(10 \log_{10}\left(\frac{1}{\delta}\right) > 10 \log_{10}(Y_f)\right) \quad (14)$$

$$= Q\left[\frac{10 \log_{10}\left(\frac{1}{\delta}\right) - m_f}{s_f}\right]. \quad (15)$$

where Q is the error function: $Q(u) = \frac{1}{2} \operatorname{erfc}\left(\frac{u}{\sqrt{2}}\right)$

The outage probability for a mobile **located at a distance \mathbf{r}** from its serving BS, taking into account shadowing can be written:

$$\mathbb{P}(\gamma < \delta) = Q\left[\frac{10 \log_{10}\left(\frac{1}{\delta}\right) - m_f}{s_f}\right]. \quad (16)$$

2.2 Fast fading and shadowing impact

2.2.1 Propagation

The power received by a mobile u depends on the radio channel state and varies with time due to fading effects (shadowing and multi-path reflections). Let P_j be the transmission power of a base station j and $X_{j,u}$ the fast fading for mobile u , the power $p_{j,u}$ received by a mobile u can be written:

$$p_{j,u} = P_j K r_{j,u}^{-\eta} X_{j,u} Y_{j,u}, \quad (17)$$

where $X_{j,u}$ is a RV representing the Rayleigh fading effects, whose pdf is $p_X(x) = e^{-x}$.

2.2.2 SINR calculation

The interference power received by a mobile u can be written:

$$p_{ext,u} = \sum_{j=1}^N P_j K r_{j,u}^{-\eta} X_{j,u} Y_{j,u}. \quad (18)$$

Considering that the thermal noise is negligible and that all BS transmit with the same power P_0 , we can write (dropping index u):

$$\gamma = \frac{r^{-\eta} X_0 Y_0}{\sum_{j=1}^N r_j^{-\eta} X_j Y_j}. \quad (19)$$

2.2.3 Outage probability

As previously, the outage probability is expressed as:

$$\mathbb{P}(\gamma < \delta) = 1 - \mathbb{P}(r^{-\eta} X_0 Y_0 > \delta (\sum_{j=1}^N r_j^{-\eta} X_j Y_j)). \quad (20)$$

The interference power received by a mobile u due to fast fading effects varies with time. As a consequence, the fast fading can increase or decrease the power received by u . We consider that the increase of interfering power due to fast fading coming from some base stations are compensated by decrease of interfering powers coming from other base stations. As a consequence, for the interfering power, only

the slow fading effect has an impact on the SINR. So, we assume that X_j can be neglected $\forall j \neq 0$ (this assumption will be validated by simulations in the next section):

Assumption: $\mathbb{P}(r^{-\eta}X_0Y_0 > \delta(\sum_{j=1}^N r_j^{-\eta}X_jY_j)) \approx \mathbb{P}(r^{-\eta}X_0Y_0 > \delta(\sum_{j=1}^N r_j^{-\eta}Y_j))$.

With this assumption, we have:

$$\begin{aligned} \mathbb{P}(\gamma < \delta) &= 1 - \mathbb{P}(X_0Y_0 > \delta \frac{1}{r^{-\eta}}(\sum_{j=1}^N r_j^{-\eta}Y_j)) \\ &= 1 - \mathbb{P}(X_0(t) > \delta Y_f) \\ &= 1 - \int_0^\infty \mathbb{P}(x > \delta Y_f) p_X(x) dx \\ &= 1 - \int_0^\infty \mathbb{P}(10 \log_{10}(\frac{x}{\delta}) > 10 \log_{10}(Y_f)) e^{-x} dx. \end{aligned}$$

As a consequence, the outage probability for a mobile **located at a distance r** from its serving BS, taking into account both shadowing and fast fading can be written:

$$\mathbb{P}(\gamma < \delta) = \int_0^\infty Q \left[\frac{10 \log_{10}(\frac{x}{\delta}) - m_f}{s_f} \right] e^{-x} dx, \quad (21)$$

where Q is the error function: $Q(u) = \frac{1}{2} \text{erfc}(\frac{u}{\sqrt{2}})$

Remark

We notice that Eq. 21 characterizes the cumulative density function (CDF) of γ . That formula also allows to analyze the coverage of a BS for a given outage probability.

3 Analytical fluid model

To calculate the outage probability we need to express analytically the factors m_f and s_f . In this aim we use the fluid model approach developed in [7]. The fluid model and the traditional hexagonal model are two simplifications of the reality. As shown in [7], none is a priori better than the other but the latter is widely used,

especially for dimensioning purposes. The key modelling step of the model consists in replacing a given fixed finite number of transmitters (base stations or mobiles) by an equivalent continuum of transmitters which are distributed according to some distribution function. We consider a traffic characterized by a mobile density and a network by a base station density ρ_{BS} . For a homogeneous network, the downlink parameter y_f (Eq. 11) only depends on the distance r between the BS and the mobile. We denote it $y_f(r)$. From [7], we have

$$y_f(r) = \frac{2\pi\rho_{BS}r^\eta}{\eta - 2} [(2R_c - r)^{2-\eta} - (R_{nw} - r)^{2-\eta}]. \quad (22)$$

where $2R_c$ represents the distance between two base stations and R_{nw} the size of the network. We notice that the factor G (Eq. 10) can be rewritten (generalizing Eq. 22, dropping r , and expressing y as a function of η) as:

$$G(\eta) = \frac{y_f(2\eta)}{y_f(\eta)^2} \quad (23)$$

The higher the standard deviation of shadowing, the higher the mean and standard deviation (see hereafter Eq. 24 and 25). This increase is however **bounded** as we can see hereafter. In a realistic network, σ is generally comprised between 4 and 8 dB.

The Eq. 7 means that the effect of the environment of any mobile of a cell, on the y_f factor, is **characterized by a function** $H(r, \sigma)$. This function takes into account the shadowing and a **topological G factor** which depends on the position of the mobile and the characteristics of the network as

- the exponential pathloss parameter η , which can vary with the topography and more generally with the geographical environment as urban or country, micro or macro cells.
- the base stations positions and number.

From Eq. 10, we have $0 < G(r, \eta) < 1$ whatever r and η . From Eq. 7, we can express:

$$y_f(r, \eta) \leq m_f \leq \frac{y_f(r, \eta)}{G(r, \eta)^{\frac{1}{2}}} \quad (24)$$

and

$$s_f^2 \leq 2\sigma^2. \quad (25)$$

For low standard deviations (less than 4 or 5 dB), we can easily deduce from Eq. 7 a low dependency of the mean value of y with σ , and the total standard deviation s_f is very close to σ . These expressions show another interesting result. We established that $0 < G < 1$. Without the topological factor G ($G = 0$), we have $H = e^{a^2\sigma^2}$ so that m_f increases indefinitely. If $G = 1$, then $H = 1$ and $m_f = \frac{1}{a} \ln(y_f)$. For each given value of η , characterizing a given type of cells or environment, the topological factor G thus *compensates* the shadowing effects.

4 Performance evaluation

In this section, we compare the figures obtained with analytical expressions (Eq. 21) to those obtained by Monte Carlo simulations. We also validate our approximations and show the limit of our formulas.

4.1 Monte Carlo simulator

The simulator assumes an homogeneous hexagonal network made of several rings around a central cell. Figure 1 shows an example of such a network with the main parameters involved in the study.

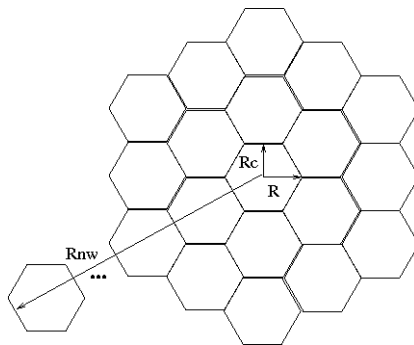


Figure 1: Hexagonal network and main parameters of the study.

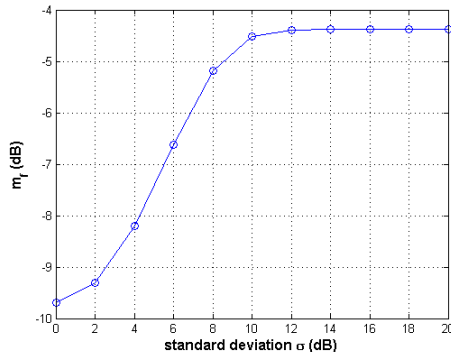


Figure 2: Variations of m_f [dB] with σ [dB] for $\eta = 3$.

The simulation consists in computing the SINR for any point u of the central cell. This computation can be done independently of the number of MS in the cell and of the BS output power (because noise is supposed to be negligible both in simulations and analytical study). At each snapshot, shadowing and fast fading RV are independently drawn between the MS and the serving BS and between MS and interfering BS. SINR samples at a given distance from the central BS are recorded in order to compute the outage probability.

The validation consists in comparing results given by simulations to the ones given by the Eq. 21.

4.2 Results

Figure 2 shows that in presence of shadowing, the value of m_f increases. However, we observe that this increase reaches a maximum value for $\sigma = 12dB$. It is due to the effect of the topology factor G which compensates the effect of shadowing. To validate our analytical approach, we present results for different pathloss factors $\eta = 3$ and 4, different standard deviations $\sigma = 3$ and 6 dB, and for mobiles located at the edge of the cell $r = R_c$ and the center of the cell $r = R_c/2$.

Figure 3 and 4 give two examples of the kind of results we are able to obtain instantaneously thanks to the formulas derived in this report for the path-loss exponents $\eta = 3$ and 4. These curves represent the outage probability for a mobile located at the edge of the cell. Figure 5 gives outage probability considering a path-

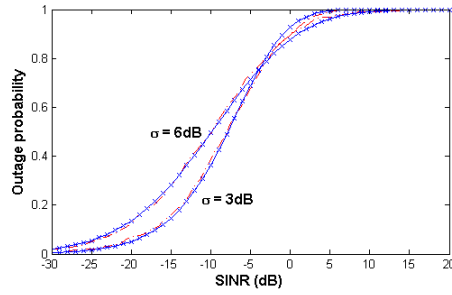


Figure 3: Outage probability for a mobile located at the edge of the cell: $r = R_c$; comparison of the fluid model (solid blue curves) with simulations (dotted red curves) on an hexagonal network with $\eta = 3$.

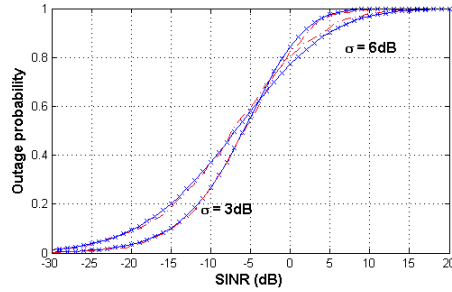


Figure 4: Outage probability for a mobile located at the edge of the cell: $r = R_c$; comparison of the fluid model (solid blue curves) with simulations (dotted red curves) on an hexagonal network with $\eta = 4$.

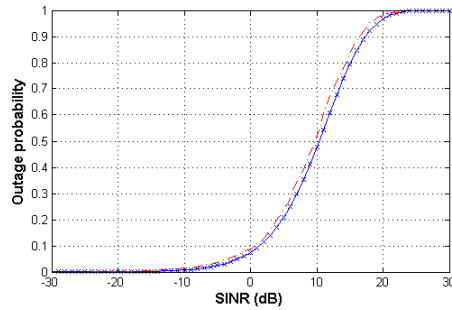


Figure 5: Outage probability for a mobile located at $r = 0.5R_c$; comparison of the fluid model (solid blue curves) with simulations (dotted red curves) on an hexagonal network with $\eta = 3$ and $\sigma = 3dB$.

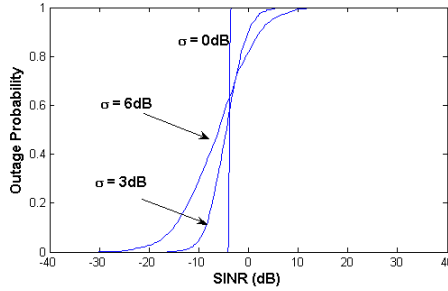


Figure 6: Outage probability for a mobile located at $r = R_c$; shadowing impact (without fast fading) for $\eta = 3$.

loss exponent $\eta = 3$, for a mobile at a distance of $r = R_c/2$. Figures 3, 4 and 5 show that our analytical approach gives results (solid blue curves) very close to the ones obtained with simulations (dotted red curves).

Figures 3, 4 5 and 6 show the specific impact of shadowing alone (Figure 6) and both shadowing and fast fading (Figures 3, 4, 5) on the outage probability. In particular the curve $\sigma = 0$ dB (Figure 6) shows that, without shadowing and fast fading, the SINR received at the edge of the cell reaches -4dB. Since there is no random variations of the propagation, this value is deterministic: the outage is 0% until a SINR of -4dB and 100% for higher values of SINR. When the shadowing is taken into account (but not the fast fading), we observe for $\sigma = 3$ dB, a SINR of -8 dB for an outage probability of 10%. And when the shadowing and the fast fading are both taken into account (Figure 3), we observe for $\sigma = 3$ dB, a SINR of -16 dB for an outage probability of 10%.

With these results, an operator would be able to analyze the outage, and to admit or reject new connections according to each specific environment of the entering MS. Indeed, the SINR depends on the mobile location and on the environment (Eq. 7), i.e. the shadowing, the exponential pathloss parameter η (which depends on different characteristics and particularly the cell dimensions and the type of environment e.g. urban or country), the number and positions of the base stations.

4.3 Applications

Hereafter we present some other examples of possible applications of the analytical approach we proposed.

4.3.1 Coverage

The coverage of a BS can be defined as the area where the SINR received by a mobile reaches a given value with a given outage probability. Figure 4 shows, for $\eta=4$, that at the edge of the cell ($r=R_c$), the SINR reaches the value of -15 dB with a probability of 10%, when $\sigma = 3dB$. With the same outage probability (10%), the SINR only reaches -20 dB when $\sigma = 6dB$.

4.3.2 Capacity

Since the capacity and the throughput D can be expressed as a function of the SINR, for example using the Shannon expression $D \propto \log_2(1 + SINR)$, the analytical method we developed can be used to analyze this performance indicator instantaneously.

5 Conclusion

In this report, we propose and validate by Monte Carlo simulations an analytical model for the estimation performances in cellular networks. We address the issue of finding an easy way to use formula for the outage probability in cellular networks, while considering pathloss, shadowing and fast fading. Taking into account pathloss and shadowing, we first express the inverse of the SINR of a mobile located at a given distance of its serving BS as a lognormal random variable. We give close form formulas of its mean m_f and standard deviation s_f based on the analytical fluid model. It allows us to analytically express the outage probability at a given distance of the serving BS. We then consider both pathloss, shadowing and fast fading and give an analytical expression of the outage probability at a given distance of the serving BS. The fluid model allows us to obtain formulas that only depend on the distance to the serving BS. Monte Carlo simulations show that our assumptions

are valid. The formulas derived in this report allow to obtain performances results instantaneously.

The proposed framework is a powerful tool to study performances of cellular networks and to design fine algorithms taking into account the distance to the serving BS, shadowing and fast fading. It can particularly easily be used to study frequency reuse schemes in OFDMA systems.

Appendix 1

Each BS transmits a power $P_j = P$ so the power received by a mobile is characterized by a lognormal distribution X_j as $\ln(X_j) \propto N(am_j, a^2\sigma_j^2)$ and we can write $m_j = \frac{1}{a}\ln(P_j r_j^{-\eta})$ (to simplify the calculation we consider $K=1$). So the total power received by a mobile is a lognormal RV X characterized by its mean and variance $\ln(X) \propto N(am, a^2\sigma_t^2)$ and we can write: $am = \ln\left(\sum_{j=0, j \neq b}^N e^{(\ln P_j - \eta \ln r_j + \frac{a^2\sigma_j^2}{2})}\right) - \frac{a^2\sigma_t^2}{2}$ so we have since $P_j = P$ whatever the base station j , considering identical $\sigma_j = \sigma$:

$$am = \left(\ln P + \frac{a^2\sigma^2}{2}\right) + \ln\left(\sum_{j=0, j \neq b}^N e^{-\eta \ln r_j}\right) - \frac{a^2\sigma_t^2}{2} \quad (26)$$

We can express the mean interference power $\overline{I_{ext}}$ received by a mobile as:

$$\ln(\overline{I_{ext}}) = \left(\ln P + \frac{a^2\sigma^2}{2}\right) + \ln\left(\sum_{j=0, j \neq b}^N r_j^{-\eta}\right) - \frac{a^2\sigma_t^2}{2} \quad (27)$$

The variance $a^2\sigma_t^2$ of the sum of interferences is written as

$$a^2\sigma_t^2 = \ln\left(\frac{\sum_j e^{(2am_j + a^2\sigma^2)(e^{a^2\sigma^2} - 1)}}{(\sum_j \exp^{am_j + \frac{a^2\sigma^2}{2}})^2} + 1\right) \quad (28)$$

Introducing

$$G(\eta) = \frac{\sum_{j=0, j \neq b}^N r_j^{-2\eta}}{\left(\sum_{j=0, j \neq b}^B r_j^{-\eta}\right)^2} \quad (29)$$

the mean value of the total interference received by a mobile is given by

$$\overline{I_{ext}} = P \sum_{j=0, j \neq b}^N r_j^{-\eta} e^{\frac{a^2\sigma^2}{2}} \left((e^{a^2\sigma^2} - 1)G(\eta) + 1 \right)^{-1/2} \quad (30)$$

and

$$a^2\sigma_t^2 = \ln\left((e^{a^2\sigma^2} - 1)G(\eta) + 1\right) + \ln\left(e^{a^2\sigma^2}\right) \quad (31)$$

Appendix 2

We denote I_{int} the power received by a mobile coming from its serving BS. Since the ratio of two lognormal RV's is also a lognormal RV, we can write the following mean and logarithmic variance:

$$m_f = \frac{\overline{I_{ext}}}{\overline{I_{int}}} \quad (32)$$

and thus, considering that: $P_b = P$, we can write, dropping the index b:

$$m_f = \frac{\sum_j r_j^{-\eta}}{r^{-\eta}} e^{\frac{a^2\sigma^2}{2}} \left((e^{a^2\sigma^2} - 1)G(\eta) + 1\right)^{-1/2} \quad (33)$$

and finally, denoting:

$$H(\sigma) = e^{\frac{a^2\sigma^2}{2}} \left((e^{a^2\sigma^2} - 1)G(\eta) + 1\right)^{-1/2} \quad (34)$$

we have

$$m_f = y_f(\eta)H(\sigma) \quad (35)$$

In dB, we can express

$$m_{f_{dB}} = \frac{1}{a} \ln(y_f(\eta)H(\sigma)) \quad (36)$$

In an analogue analysis, the standard deviation is given by $a^2s_f^2 = a^2\sigma_t^2 + a^2\sigma^2$ so we have

$$a^2s_f^2 = 2(a^2\sigma^2 - \ln(H(\sigma))) \quad (37)$$

References

- [1] L.Fenton "The sum of lognormal probability distributions in scatter transmission system", IEEE (IRE) Transactions on Communications, CS-8, 1960

- [2] S. Shwartz and Y.S. Yeh, On the distributions functions and moments of power sums with lognormal components, Bell Syst, Tech J, vol 61, pp 1441 1462, Sept 1982
- [3] G.L. Stuber, principles of Mobiles Communications, Norwell , MA. Kluwer, 1996
- [4] Adnan A. Abu-Dayya and Norman C. Beaulieu, Outage Probabilities in the Presence of Correlated Lognormal Interferers, IEEE Transactions on Vehicular Technology, Vol. 43, N 1, February 1994
- [5] J.-M. Kelif, Admission Control on Fluid CDMA Networks, Proc. of WiOpt, Apr. 2006.
- [6] J-M. Kelif and E. Altman, Downlink Fluid Model of CDMA Networks, Proc. of IEEE VTC Spring, May 2005.
- [7] J-M Kelif, M. Coupechoux and P. Godlewski, Spatial Outage Probability for Cellular Networks, Proc. of IEEE Globecom Washington, Nov. 2007
- [8] C. C. Chan and Hanly, Calculating the Outage Probabability in CDMA Network with Spatial Poisson Traffic, IEEE Trans. on Vehicular Technology, Vol. 50, No. 1, Jan. 2001.
- [9] S. E. Elayoubi and T. Chahed, Admission Control in the Downlink of WCDMA/UMTS, Lect. notes comput. sci., Springer, 2005.
- [10] J. S. Evans and D. Everitt, Effective Bandwidth-Based Admission Control for Multiservice CDMA Cellular Networks, IEEE Trans. on Vehicular Technology, Vol. 48, No. 1, Jan. 1999.
- [11] M. Pratesi, F. Santucci, M. Ruggieri, Outage Analysis in Mobile Radio Systems with Generically Correlated Log-Normal Interferers, IEEE Trans. on Communications, Vol. 48, No. 3, Mar. 2000.
- [12] J.-P. M. G. Linnartz, Exact Analysis of the Outage Probability in Multiple-User Mobile Radio, IEEE Trans. on Communications, Vol. 40, No. 1, Jan. 1992.

- [13] V. V. Veeravalli and A. Sendonaris, The Coverage-Capacity Tradeoff in Cellular CDMA Systems, *IEEE Trans. on Vehicular Technology*, Vol. 48, No. 5, Sept. 1999.
- [14] G. Immovilli and M. L. Merani, Simplified Evaluation of Outage Probability for Cellular Mobile Radio Systems, *IEEE Electronic Letters*, Vol. 27, No. 15, Jul. 1991.