TD INF567

Wireless Propagation

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1 Path-loss models

A set of radio measurements are performed in a urban area. A transmitter emits at a power of $P_t = 46$ dBm and carrier frequency $f_c = 900$ MHz. The transmitter antenna gain is 16 dBi and the receiver antenna gain is 0 dBi. Received power is measured at various distances from the transmitter and is shown in Table 1. We want to adapt the simplified path-loss model to these measurements. We assume a reference distance of $d_0 = 1$ m.

Table 1: Measurements.

Distance d [m]	Received power [dBm]
50	1.2
100	-10.4
250	-19.8
500	-30.4
1000	-50.4

Question 1 *We assume that K is the same as for the free-space model at distance d_0. Compute K in linear scale and in dB.*

Question 2 *Compute the path-loss exponent that minimizes the mean square error between the simplified model and the measurements (in dBm). Compute the received power in dBm at 750 m.*

We now evaluate the shadowing standard deviation σ.

Question 3 *Compute the variance of the measurements relatively to the simplified model found in the previous question. Deduce the shadowing standard deviation.*

A carrier frequency of $f_c = 900$ MHz is typical for GSM. Next generation cellular networks are expecting to use millimeter waves, i.e., carrier frequency between 30 and 300 GHz, due to the huge amount of available spectrum in these bands. Let us consider a carrier frequency of $f_c = 30$ GHz. Assume that the path-loss exponent at this frequency is about 2.5. We assume that beamforming with N directional antennas brings a maximum gain of $10 \log_{10} N$ on the received power. We now assume that $G_t = 30$ dBi and $G_r = 5$ dBi.

Question 4 *Compute the new value of K. How many antennas are required in mm-wave to received the same power at a distance of 750 m as with a signal at 900 MHz, assuming that the transmit power remains the same?*

Antenna gains in dBi are defined with reference to the isotropic radiator, a non-existent ideal antenna with omnidirectional radiation. A simple realistic antenna is the $\lambda/2$-dipole, which has a gain of 2.15 dBi and whose length is precisely $\lambda/2$. Base stations directional antennas can be realized by combining in the same radome N_d such dipoles. A gain of $10 \log_{10}(N_d)$ can then be achieved.

Question 5 *How many $\lambda/2$-dipoles are required for a directional antenna with gain 16 dBi? In a GSM radome, we organize dipoles in 2 columns. What is the height of a radome at 900 Mhz? In a mm-wave panel using beamforming, we organize dipoles in square. What is the height of a square panel of $N * N_d$ dipoles at 30 GHz?*

2 Cell radius

We consider a cellular network using a carrier frequency $f_c = 2.6$ GHz and a signal bandwidth of $W = 20$ MHz (typical of LTE in France). We use the simplified path-loss model with reference distance $d_0 = 1$ m and K computed at this distance with the free-space model and a path-loss exponent $\alpha = 3.5$. We assume a shadowing standard deviation of $\sigma = 8$ dB. The base station transmits at $P_t = 46$ dBm and has an antenna gain of 19 dBi. We ignore co-channel interference (the cell is isolated). We would like to ensure a physical data rate of 10 Mbps 99 % of the time at cell edge. Assume that the Shannon formula is achievable. We define: $Q(z) \triangleq \int_z^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$.

Question 6 *Compute the maximum cell size.*

We now want to evaluate the expected cell coverage area. Assume that the transmit power is set so that at cell edge (at a distance R), the average received power is $\bar{P}$, i.e., $\bar{P} = P_t + K_{dB} - 10\alpha \log(R)$. The minimum required received power for coverage is denoted P_r^*.

Question 7 *Preliminary result: Define $C \triangleq \frac{2}{R^2} \int_0^R rQ\left(a + b \ln \frac{r}{R}\right) dr$. Show that $C = Q(a) + \exp\left(\frac{2-2ab}{b^2}\right) Q\left(\frac{2-ab}{b}\right)$ by using integration by parts and setting $k = a + b \ln \frac{r}{R}$.*

Question 8 *Compute the proportion of the cell area that is covered. Numerical application: use numerical values of the previous question, set $P_r^* = -110$ dBm, and set $\bar{P}$ such that the outage probability at cell edge is $P_{out}^{edge} = 10\%$.*

3 SIR and SINR

We consider a infinite linear cellular network (along a highway for example) made of cells of radius R. Assume that the path-gain model at a distance r is of the form $g(r) = Kr^{-\eta}$.

Question 9 *Compute the Signal to Interference Ratio (SIR) at a distance r on the downlink. Show that the SIR does not depend on the transmit power and only depends on the relative distance of the receiver towards the cell edge.*

Let now consider an infinite 2D cellular network. We assume that cells are non-overlapping tangent disks of same radius R that pave the 2D plane, so that the minimum inter-base station distance is $2R$. Let $\rho = 1/(\pi R^2)$. We consider a simple path-loss model, where the received power $p_r(r)$ at a distance r can be written as a function of the transmit power p_t as follows: $p_r(r) = p_t K r^{-\eta}$, where K is a constant and $\eta > 2$ is the path-loss exponent.

Question 10 *Show that the SINR γ can be approximated as:*

$$\gamma = \frac{r^{-\eta}}{\frac{2\pi\rho}{\eta-2}(2R-r)^{2-\eta} + \frac{N}{P_t K}},$$

where N is the noise power. Hint: Assume we have a continuum of interfering base stations and compute the contribution of an elementary interfering surface $zdzd\theta$.