CDMA Cellular Access and 3G

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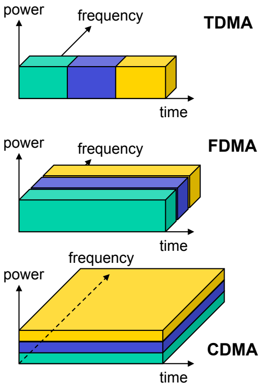
Outlines

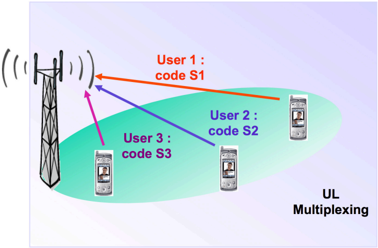
- Introduction
- Spread spectrum
- Channelization and scrambling
- Power control
- Outage probability
- Blocking probability

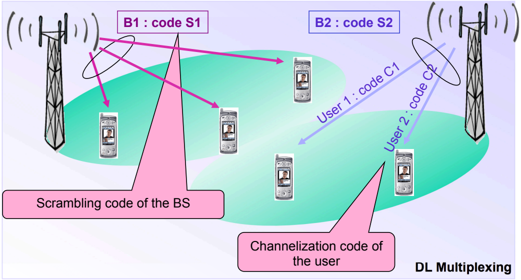
Introduction : 3G multiplexing I

Recall the different multiplexing schemes :



Introduction : 3G multiplexing II



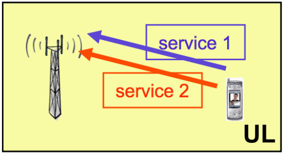


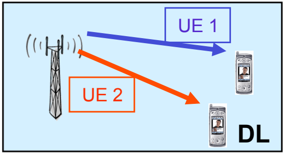
- All transmitters transmit all the time on the whole system bandwidth (duplexing is FDD)
- UL : users use different scrambling codes and have a channelization code per service
- DL : every BS has a scrambling codes, users have different channelization codes
- Spreading code = scrambling code × channelization code

Introduction : 3G multiplexing III

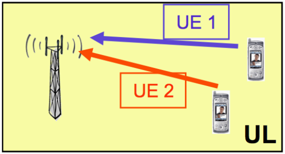
Spreading codes are obtained by combining (i.e. multiplying) :

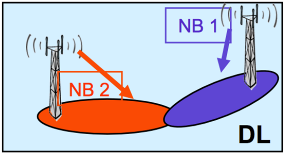
- Channelization codes :



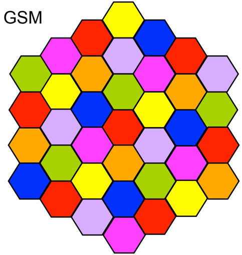


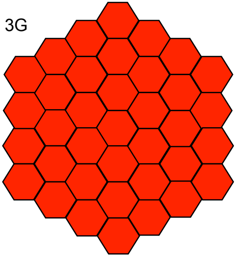
- Scrambling codes :





Introduction : 3G planning



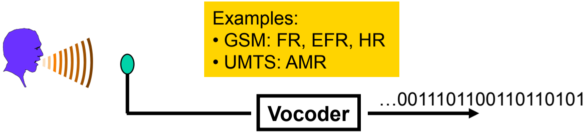


- GSM : frequency reuse with $K > 1$ (typically 3 with frequency hopping, 9 without frequency hopping or even higher for BCCH)
- 3G : $K = 1$, codes are planned, easier planning (there are many such codes, e.g. 504 in UMTS), stronger interference

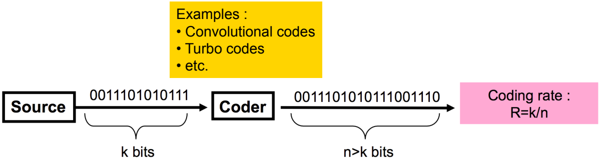
Introduction : Other codes

Other kinds of codes :

- Source coding :



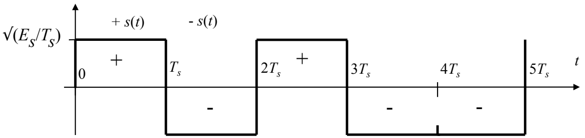
- Channel coding :



Spread spectrum : Modulations

Remember :

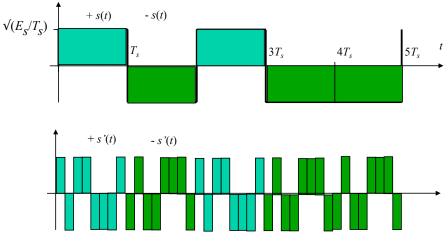
- Numerical information : $\mathbf{x} = \{x_0, x_1, ..., x_k, ...\}$ with $x_k \in A$ where A is the signal constellation, e.g., in BPSK $A = \{+1; -1\}$
- A symbol is transmitted using a waveform $s(t)$ on the interval $[kT_s; (k+1)T_s]$ as $x_k s(t - kT_s)$, where T_s is the symbol duration
- An impulsion train is $X(t) = \sum_k x_k s(t - kT_s)$
- Symbol rate is $R_s = 1/T_s$, bit rate is $R_b = \log_2(|A|)/T_s$, signal bandwidth is approximately $W \approx 1/T_s$
- In BPSK/antipodal modulation, "0" is coded +1 and "1" is coded -1



- In QPSK, Q and I are used and $A = \{1+i; 1-i; -1+i; -1-i\}$

Spread spectrum : Principle I

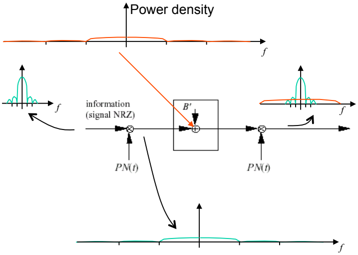
Direct sequence spread spectrum :



- Spread spectrum consists in choping/hashing the signal by a new waveform $s'(t)$, which is also a sequence
- The new signal is still antipodal with same energy per bit (in particular, performance in BER is the same)
- New elementary symbols are called *chips* and the number of chips per QAM symbol is called the spreading factor, e.g., here $n = 8$

Spread spectrum : Principle II

Spectral vision :



- Narrow band signal is spread over a wide band after the multiplication by the spreading code (here : PN for Pseudo-Noise sequence). UMTS spreads the signal over 5 MHz vs. 1.25 MHz in cdmaOne, hence the term Wideband CDMA (WCDMA)
- The spreading code is designed such that statistics of the interference are similar to white gaussian noise, interference is "whitened"

Spread spectrum : Matched filter I

- Assume a perfect channel (no noise, no interference, unit gain). Received signal is

  $$r(t) = s(t - \tau_0) \times PN(t - \tau_0)$$

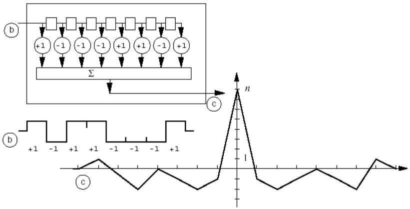
  The receiver multiplies by the (known) spreading sequence :
  $r(t) \times PN(t - \tau_0) = s(t - \tau_0)$ because $PN(t - \tau_0) \times PN(t - \tau_0) = 1$
- Assume now white gaussian noise n and interference PN' :

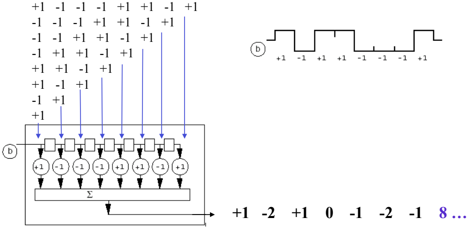
  $$r(t) = s(t - \tau_0) \times PN(t - \tau_0) + PN'(t - \tau_1) + n(t)$$

  The receivers performs a correlation between the received signal and PN :
  $\langle r(t), PN(t - \tau_0)\rangle =$
  $s(t - \tau_0)\langle PN(t - \tau_0), PN(t - \tau_0)\rangle + \langle PN'(t - \tau_1), PN(t - \tau_0)\rangle + \langle n(t), PN(t - \tau_0)\rangle$
  The second term is small if PN and PN' are designed with low cross-correlation

Spread spectrum : Matched filter II

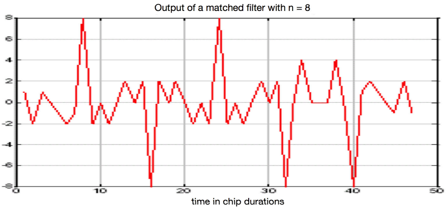
Example : take the sequence $s'(t) = + - + + - - - +$

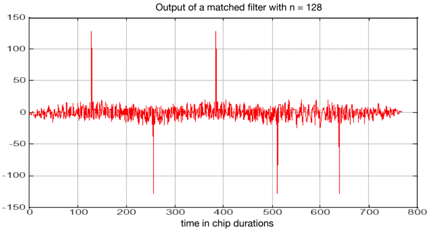




Spread spectrum : Matched filter III

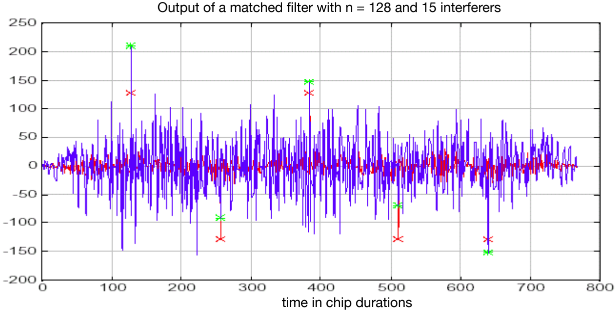
Examples with $n = 8$ and $n = 128$:





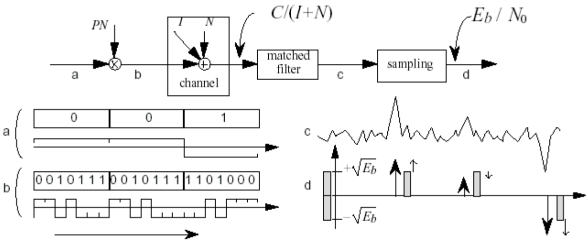
Spread spectrum : Matched filter IV

Example with $n = 128$ and 15 interferers : at bit instants, the value is correct



Spread spectrum : Matched filter V

Simplified transmission and reception chain :



Simplified transmission and reception chain in CDMA

Note : in this picture N_0 represents (despite the name) the spectral power density of the noise plus interference

Spread spectrum : Energy budget

Spreading factor (aka spreading gain, aka CDMA processing gain) :

- We have : $\frac{S}{N} = \frac{E_c}{N_0}$ and $n = \frac{T_b}{T_c} = \frac{R_c}{R_b}$, where R_c is the chip rate
- Power is the same before and after spreading, so that :

$$\frac{E_b}{T_b} = \frac{E_c}{T_c},$$

which is equivalent to $E_b = nE_c$

- As a consequence :

$$\frac{S}{N} = \frac{1}{n}\frac{E_b}{N_0} = \frac{R_b}{R_c}\frac{E_b}{N_0}$$

In dB :

$$\left(\frac{S}{N}\right)_{dB} = \left(\frac{E_b}{N_0}\right)_{dB} - G_e,$$

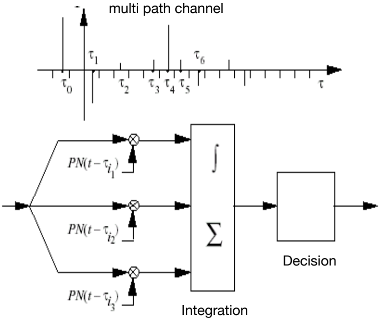
where

$$G_e = 10\log\frac{R_c}{R_b}$$

is the *spreading factor*

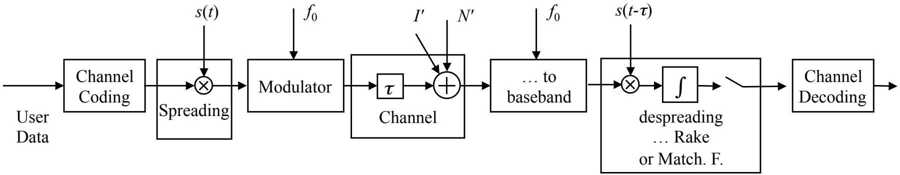
Spread spectrum : Rake receiver I

- The matched filter is not sufficient in case of multi-path propagation
- A *Rake* receiver is used with several correlators (*fingers*, typically 4), one for every most dominant paths
- In soft-handover, correlators are configured with different spreading codes

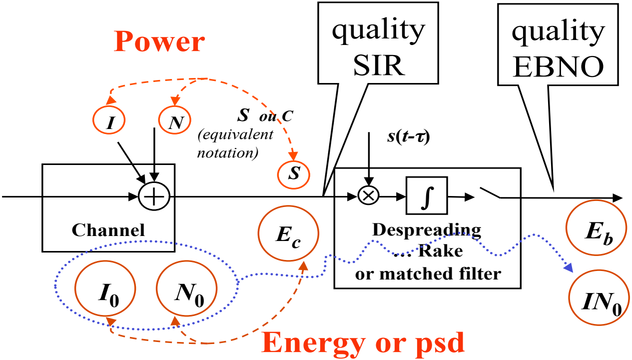


Spread spectrum : Rake receiver II

New simplified transmission and reception chain with Rake receiver :



Spread spectrum : Radio quality



- SIR is a power ratio at the entrance of the reception chain
- Eb/N0 is an energy ratio after despreading
- In "Eb/N0", remember that N0 represents the psd of noise plus interference

Channelization I

Hadamard matrices :

- Notation : we consider an antipodal modulation, "0" is represented by $+1$ or $+$ and "1" by -1 or simply -
- Hadamard matrices are defined recursively : $H_1 = [+1]$ and

$$H_{2N} = \begin{bmatrix} +H_N & +H_N \\ +H_N & -H_N \end{bmatrix}$$

- Examples :

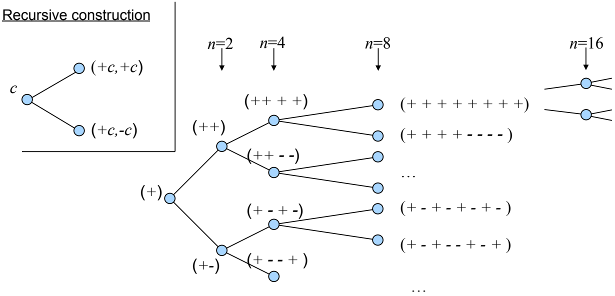
$$H_4 = \begin{bmatrix} + & + & + & + \\ + & - & + & - \\ + & + & - & - \\ + & - & - & + \end{bmatrix}$$

  H_{64} and H_{128} are used in IS95, H_{256} (UL) and H_{512} (DL) in 3G

- Remarks : 1) matrix lines are orthogonal, 2) orthogonality is lost if one line is shifted wrt another, 3) lines are still orthogonal if we multiply every line by a constant

Channelization II

Another representation is a tree :

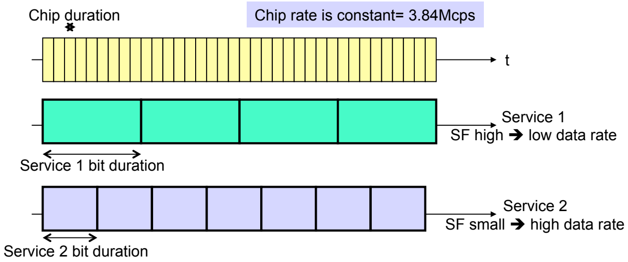


- The lines of a Hadamard matrix are the physical channels of 3G (they corresponds to the slots in GSM). One line of the matrix is called a Walsh code
- Generally, a cell cannot use all the lines of the matrix, the cell would be "overloaded" with too much interference. The system is limited by the quality rather by the number of codes

Channelization III

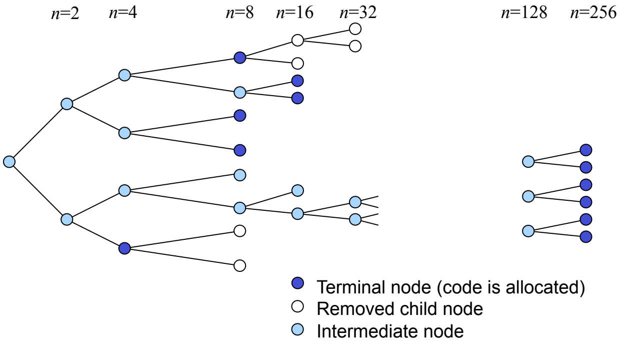
Variable spreading factor :

- UMTS allows for a variable spreading factor depending on the data rate required for the service. Hence the "multi-rate", "multi-service" feature of 3G. These codes are Orthogonal Variable Rate Spreading Factor Codes (OVSF).



Channelization IV

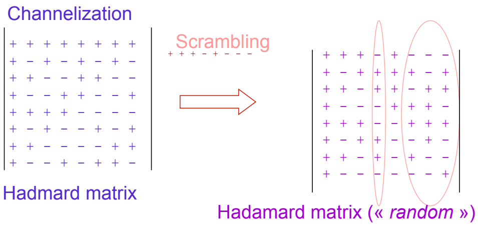
OVSF allocation must follow certain rules in order to preserve orthogonality :



- All child codes cannot be allocated
- All intermediate codes between an allocated code and the root cannot be allocated

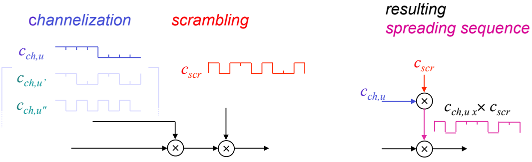
Scrambling I

- The lines of an Hadamard matrix are not (or don't seem) random
- Scrambling codes are used to "randomized" these lines and have good cross-correlation properties
- IS95 uses m-sequences : the system is synchronized (with GPS), so that the sequences used by the BSs/MSs can be deduced from a unique long sequence and offsets
- UMTS/WCDMA uses Gold sequences : the system is not synchronized, different sequences have to be used by different equipments



Scrambling II

Computation of the spreading sequence :



Alternative representation

$c_{ch,u}$ 00001111 c_{scr} 01001011…

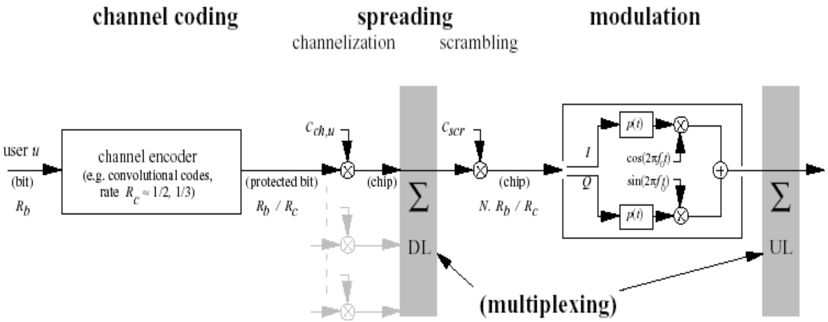
$c_{ch,u'}$ 00110011

$c_{ch,u''}$ 01010101

00001111
⊕01001011
01000100

Scrambling III

Simplified transmission chain with DL and UL multipexing :



- Remarks : 1) there is a single scrambling code per transmitter, 2) on the UL, multiplexing is done over the air

Power control : Model

Assumptions :

- Uplink CDMA network
- Single service case : all users have the same bit rate and Eb/N0 (ϵ^*) or SINR (γ^*) requirement.
- User u is active w.p. $P[\nu_u = 1] = p$. The number of users is Poisson distributed with parameter λ. Overall the number of active users is Poisson distributed with parameter $p\lambda$.
- Path-gain between transmitter u and receiver v is $g(u, v)$.
- MS u transmits at power $P_u \leq P_u^{max}$ and is received with power S_u.
- Interference : We denote $I = I_{ex} + I_{in}$ and $IN = I + N$, where I_{ex} is the other-cell interference, I_{in} is the total inner-cell power (incl. useful signal) and N is the noise power.

Power control based on signal level I

- A simplifying assumption to allow tractability.
- All MSs target a common received power : $\forall u, S_u = S^*$ (i.e. S_u is constant).
- Recall that $S^* = P_u g(u, b)$, where b is the serving BS of user u.
- This power control is possible for u iff : $P_u = S^*/g(u, b) \leq P_u^{max}$.
- Imperfect power control model : $S_u = S^* X/E[X]$, where $10\log(X)$ is normally distributed with mean 0 and variance σ_{PC}^2.

Power control based on signal quality I

- This power control is implemented in UMTS using an iterative algorithm.
- All MSs target a common SINR : $\forall u, S_u/(IN - \nu_u S_u) = \gamma^*$ (i.e. SINR is constant).
- Recall that $IN = \sum_v \nu_v P_v g(v, b) + N$.
- This power control is possible iff the following system has a solution in the variables $\{P_u\}$:

$$\begin{aligned} \frac{P_u g(u, b)}{\sum_{v \neq u} \nu_v P_v g(v, b) + N} &= \gamma^* \quad \forall u & (1) \\ P_u &\leq P_u^{max} & (2) \end{aligned}$$

- Imperfect power control model : $S_u/IN = \gamma^* X/E[X]$, where $10 \log(X)$ is normally distributed with mean 0 and variance σ_{PC}^2.

Power control based on signal quality II

- Assuming perfect power control and U users, the system has a matrix form :

$$(Id - F)P = w, \tag{3}$$

where Id is the identity matrix,
$P = [P_u]_{1 \leq u \leq U}$ is the column vector of users transmit powers,

$$w = \gamma^* N \left[\frac{1}{g(1, b(1))} \cdots \frac{1}{g(U, b(U))} \right]^T$$

and

$$\begin{aligned} F_{u,v} &= \gamma^* \nu_v \frac{g(v, b(u))}{g(u, b(u))} \quad \text{if } u \neq v \\ &= 0 \text{ otherwise} \end{aligned} \tag{4}$$

- This system has a solution iff the spectral radius of F, ρ_F (i.e. the maximum modulus of its eigenvalues), is < 1. In this case : $P = (I - F)^{-1} u$.

Power control based on signal quality III

Single cell analysis :

- There a single BS : $\forall u, g(u, b(u)) = g(u, b)$
- With this simplification we can solve the power control system as follows :

$$P_u = \frac{I_b + N}{g(u,b)} \frac{\gamma^*}{1 + \nu_u \gamma^*},$$

where

$$\begin{aligned} I_b &= \sum_v \nu_v P_v g(v, b) \\ &= \frac{N \sum_u \frac{\nu_u \gamma^*}{1+\nu_u \gamma^*}}{1 - \sum_u \frac{\nu_u \gamma^*}{1+\nu_u \gamma^*}} \end{aligned} \quad (5)$$

provided that $x_{UL} = \sum_u \frac{\nu_u \gamma^*}{1+\nu_u \gamma^*} < 1$

- x_{UL} is called the *uplink radio load*.

Power control based on signal quality IV

Multi-cell analysis :

- There is no tractable way of analyzing the spectral radius of F
- It is usual to consider the other-cell interference factor (OCIF) $f = I_{ex}/I_{in}$. In this case, the feasibility condition is :

$$(1+f)\sum_{u\in b}\frac{\nu_u\gamma^*}{1+\nu_u\gamma^*} \quad < \quad 1.$$

- We can check by simulations that f is approximately independent of the number of users if the traffic is homogeneous.
- For $x_{UL} = 1$ and $\nu_u = 1, \forall u$, we obtain the pole capacity :

$$N_{UE}^{pole} = \left\lfloor \frac{1}{1+f}\frac{1+\gamma^*}{\gamma^*} \right\rfloor$$

- In practice, because of channel variations, we impose $x_{UL} \leq x_{UL}^{max}$ with $x_{UL}^{max} = 0.5$ or 0.65 (for typical values).

Outage probability for PC based on signal level I

- Computing the outage probability is equivalent to finding the cdf of I :

$$\begin{aligned} P_{out} &= P\left[\frac{S^*}{I+N-S^*} < \gamma\right] \\ &= P\left[\frac{S^*}{I+N} < \gamma'\right] \\ &= P\left[I > \frac{S^*}{\gamma'} - N\right] \\ &= P[I > I_{thresh}], \end{aligned}$$

where γ^* is a threshold SINR and $\gamma' \triangleq \frac{\gamma^*}{1+\gamma^*}$.

Outage probability for PC based on signal level II

- Assuming perfect PC, **inner-cell interference** is as follows :

$$I_{in} = 1_{\{N_{UE} \neq 0\}} \sum_{u=1}^{N_{UE}} \nu_u S^* = \sum_{u=0}^{N_{UE}} \nu_u S^*$$

where N_{UE} is Poisson distributed with parameter λ and $\nu_0 = 0$.

- We have : $E[I_{in}] = S^* p\lambda$, $E[I_{in}^2] = S^{*2} p\lambda(1 + p\lambda)$, $Var[I_{in}] = S^{*2} p\lambda$.

Outage probability for PC based on signal level III

- **Other-cell interference** : We have a generic formula (w.r.t reference BS b), but not the distribution :
$$I_{ex} = \sum_{v \notin b(0)} \frac{S^* g(v, b)}{g(v, b(v))},$$
where $b(v)$ is the serving BS of v.
- We assume that there are constants $f = E[I_{ex}]/E[I_{in}]$, called other-cell interference factor, and $f' = Var[I_{ex}]/Var[I_{in}]$ (obtained by simulations).
- I_{ex} is approximated by a Gaussian variable (CLT) with $E[I_{ex}] = fS^* p\lambda$ and $Var[I_{ex}] = f' S^{*2} p\lambda$.

Outage probability for PC based on signal level IV

Outage probability under Gaussian approximation :

- Interference is approximated by a Gaussian variable :

$$P_{out} = Q\left(\frac{I_{thresh} - (1+f)S^* p\lambda}{S^*\sqrt{(1+f')p\lambda}}\right).$$

- If we introduce the uplink radio load $x_{UL} = E[I]/E[IN]$, then $N = E[I](1-x_{UL})/x_{UL}$ and the outage probability can be re-written :

$$P_{out} = Q\left(\frac{\frac{1}{\gamma'} - \frac{(1+f)p\lambda}{x_{UL}}}{\sqrt{(1+f')p\lambda}}\right).$$

- This expression can be inverted to obtain the mean number of users per cell :

$$\lambda = \frac{x_{UL}}{\gamma' p(1+f)} + \frac{B^2 x_{UL}^2(1+f')}{2p(1+f)^2}\left(1 - \sqrt{1 + \frac{4(1+f)}{\gamma' x_{UL} B^2(1+f')}}\right),$$

where $B = Q^{-1}(P_{out})$.

Blocking for PC based on signal quality I

- UMTS is a multi-service technology based on dedicated physical channels.
- Remember that for a single service, $x_{UL} \leq x_{UL}^{max}$ and $\nu_u = 1\ \forall u$, we have $N_{UE} \leq N_{UE}^{max}$ with $N_{UE}^{max} = \left\lfloor \frac{x_{UL}^{max}}{1+f} \frac{1+\gamma^*}{\gamma^*} \right\rfloor$.
- There are thus N_{UE}^{max} channels and blocking probability can be obtained using Erlang-B.
- When multi-services are considered, every user (or service) has its own target SINR γ_u^* and the condition on the uplink radio load can be written :

$$(1+f)\sum_{u \in b} \frac{\nu_u \gamma_u^*}{1+\nu_u \gamma_u^*} \quad < \quad x_{UL}^{max}$$

- Every user u contributes with weight $w_u = (1+f)\frac{\nu_u \gamma_u^*}{1+\nu_u \gamma_u^*}$ to the total load.

Blocking for PC based on signal quality II

Multi-Erlang-B :

- Consider N services. Service i requires w_i circuits and we denote $\mathbf{w} = (w_1, ..., w_N)$. There are W circuits in total.
- Service i call arrivals are Poisson with parameter λ_i and have exponential durations with parameter μ_i. The load of service i is denoted $\alpha_i = \lambda_i/\mu_i$. The total load is $\rho = \frac{1}{W}\sum_i \alpha_i w_i$.
- Let $\mathbf{n} = (n_1, ..., n_N)$ be the vector of the numbers of active calls of all services. $\mathbf{n}(t)$ is a Markov process with state space $\{\mathbf{n}|\mathbf{n}\cdot\mathbf{w} \leq W\}$.
- Stationary probabilities are given by :

$$\pi(\mathbf{n}) = \pi(0)\frac{\alpha_1^{n_1}}{n_1!}\cdots\frac{\alpha_N^{n_N}}{n_N!}.$$

- The blocking probability of service i is given by :

$$P_{bi} = \sum_{\mathbf{n}:W-w_i<\mathbf{n}\cdot\mathbf{w}\leq W} \pi(\mathbf{n}).$$

Blocking for PC based on signal quality III

Kaufman-Roberts :

- A recursive formula is generally used to simplify the computation of the blocking probability.
- Kaufman-Roberts approach consists in focusing on the law of occupancy of the resource rather than on the distribution of every service :

$$f(m) = \sum_{\mathbf{n}:\mathbf{n}\cdot\mathbf{w}=m} \frac{\alpha_1^{n_1}}{n_1!} \cdots \frac{\alpha_N^{n_N}}{n_N!}.$$

We have the following recursion :

$$f(m) = \frac{1}{m} \sum_{i=1}^{N} \alpha_i w_i f(m - w_i).$$

- From which we deduce blocking probabilities :

$$P_{bi} = \frac{\sum_{W-w_i<m\leq W} f(m)}{\sum_{0\leq m\leq W} f(m)}.$$

Acronyms I

BPSK	Binary Phase Shift Keying
BS	Base Station
CDMA	Code Division Multiple Access
CDF	Cumulative Distribution Function
CLT	Central Limit Theorem
DL	Downlink
FDD	Frequency Division Duplex
FDMA	Frequency Division Multiple Access
GPS	Global Positioning System
GSM	Groupe Spécial Mobile
MS	Mobile Station
NB	Node-B
OCIF	Other-Cell Interference Factor
OVSF	Orthogonal Variable Rate Spreading Factor
PC	Power Control
PN	Pseudo-Noise
PSD	Power Spectral Density
QAM	Qadrature Amplitude Modulation
QPSK	Quadrature Phase Shift Keying
SINR	Signal to Interference plus Noise Ratio
SIR	Signal to Interference Ratio
TDMA	Time Division Multiple Access
UE	User Equipment
UMTS	Universal Mobile Telecommunications System
UL	Uplink
WCDMA	Wideband Code Division Multiple Access