ROBUST BARRIER COVERAGE IN THE INTERNET OF THINGS

MARCEAU COUPECHOUX

1. Introduction

The concept of "Internet of Things" refers to the massive deployment of sensors in our daily life as well as in various economic sectors such as autonomous vehicles, robotics, agriculture, etc. These sensors, unless powered by a renewable energy, are equipped with batteries with several years of lifetime. They can be controlled by a central unit through a wireless communication protocol.

In this project, we study an application of the Internet of Things, which consists in deploying sensors to detect the intrusion of an animal or a human through a strip (or rectangular) area. This type of deployment has applications in wildlife monitoring or security. Every sensor has a sensing range, within which the presence of a moving object is detected. As the detection may fail, we require a robust deployment by adding some redundancy. We say that a set of sensors provides a robust K*-barrier coverage* if any intrusion can be detected by at least K sensors (see Figure 1).

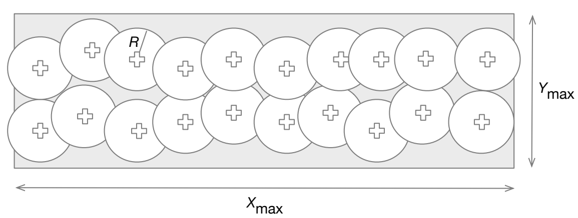

Figure 1. Sensor deployment over a strip area providing a $K = 2$ barrier coverage.

As the sensor lifetime is not infinite, we would like also to activate or desactivate sensors in order to save energy. Our objective is thus to find for a given sensor deployment, a given set of remaining lifetimes and a given K, a schedule of the sensors so that the network provides a K-barrier coverage at every instant and so that the network lifetime is maximized.

This subject as well as test files are available on this page:
`https://marceaucoupechoux.wp.imt.fr/enseignement/english-inf421-pi/`

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2. Preliminaries

In this project, $\Gamma = (W, F, c)$ is an undirected graph with node capacities, and $G = (V, E, w)$ and $G' = (V', E', w')$ are directed graphs with edge capacities. A *sensor network* is a tuple $(\mathcal{N}, R, c)$, where $\mathcal{N}$ is the set of sensors, R is the sensing range of the sensors, and $c(u) \in \mathbb{N}^*$ is the remaining lifetime of sensor $u \in \mathcal{N}$. A sensor can be activated (it switches from a OFF to a ON state) or desactivated (from ON to OFF). If a sensor is ON during n units of time, its remaining lifetime is decreased by n.

An instance of the K barrier coverage problem is a tuple $(\mathcal{N}, R, c, K, \mathcal{A})$, where $\mathcal{A} = [0, X_{max}] \times [0, Y_{max}]$ is the protected strip area.

From a sensor network $(\mathcal{N}, R, c, \mathcal{A})$, we construct a *coverage graph* $\Gamma = (W, F, c)$ as follows. The set W consists of one vertex per sensor node and two additional vertices s and t at the boundary of the area. There is an edge in F between two vertices corresponding to two sensors if their distance is less than $2R$. Additionally, there is an edge between a vertex corresponding to a sensor with s (resp. t) if its distance to the left (resp. right) boundary is less than R. Every node u has a capacity $c(u)$ and we set $c(s) = c(t) = \infty$.

Solving the K barrier coverage problem of the sensor network is equivalent to find a schedule of the nodes such that at any instant there are K node-disjoint paths from s to t in the coverage graph and such that the network lifetime is maximized.

Let now consider a directed graph $G = (V, E, w)$, where V is the set of vertices, E is the set of edges and $w : E \mapsto \mathbb{N}$ is a capacity function with *integral* values. Let $s, t \in V$ be respectively a source and a sink.

Definition 1 (s-t flow)**.** *An s-t flow in G is a mapping $f : E \mapsto \mathbb{R}^+$ such that:*

- $\forall u \in V - \{s, t\}$, $\sum_{v \in V} f(v, u) = \sum_{v \in E} f(u, v)$
- $\forall u, v \in V$, $0 \leq f(u, v) \leq w(u, v)$

The value of the s-t flow is defined as: $|f| = \sum_{v \in V} f(s, v) - \sum_{v \in V} f(v, s)$.

Let $U = \max\{w(u, v); u, v \in V\}$ be the maximum capacity of any edge in G. The *maximum-flow problem* consists in sending the maximum flow from s to t while meeting the capacity constraints on every edge and the flow conservation constraints at every vertex except s and t. Let ν^* be the maximum s-t flow value.

Definition 2 (Elementary K-flow)**.** *An elementary K-flow is a flow of one unit along K edge-disjoint paths from s to t.*

Definition 3 (K-route flow)**.** *A K-route flow is any flow from s to t that can be expressed as a nonnegative linear sum of elementary K-flows. The value of a K-route flow is equal to the total amount of flow entering the sink node t. The value of a K-flow is the associated nonnegative factor in the sum.*

Definition 4 (K-route flow problem)**.** *For a directed graph $G = (V, E, w)$ with integral edge capacities, a source $s \in V$, a sink $t \in V$, the K-route flow problem consists in finding the maximum s-t flow which can be decomposed into a nonnegative linear sum of elementary K-flows.*

Figure 2 shows an example of directed graph G (a), a K-route flow of value $\nu = 8$ with $K = 2$ (c), and its decomposition into two K-flows of respective values 1 and 3 (d-e).

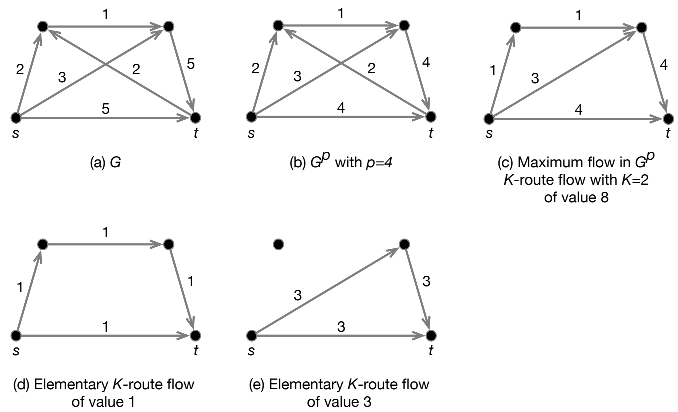

FIGURE 2. A directed graph G, a maximum K-route flow and its decomposition in elementary K-flows.

Assume now that each directed edge e of E has a *demand* $d(e) \leq w(e)$. We require from a flow f to satisfy $d(e) \leq f(e) \leq w(e)$ at every edge. Finding a feasible flow can be obtained as follows. Construct a new graph $G' = (V', E', w')$ from G by adding a new source s' and a new target t' and defining the new capacity function as:

- $\forall v \in V$, $w'(s', v) = \sum_{u \in V} d(u, v)$ and $w'(v, t') = \sum_{u \in V} d(v, u)$
- For each edge $(u, v) \in E$, $w'(u, v) = w(u, v) - d(u, v)$.
- $w'(t, s) = \infty$.

Lemma 1. *For any saturating s'-t'-flow $f' : E' \mapsto \mathbb{R}$ of G', $f = f'|_E + d$ is a feasible s-t-flow of G.*

3. Homogeneous Lifetimes

In this section, we assume that the lifetime of every sensor is 1 ($\forall u \in \mathcal{N}$, $c(u) = 1$). Let M be the maximum number of node-disjoint paths between s and t. In the following, we assume that $K \leq M$.

3.1. Maximum Number of Node-Disjoint Paths.

We have the following result, for which we omit the proof.

Lemma 2 (Upper Bound). *The maximum time for which the sensor network can provide K-barrier coverage is at most M/K.*

Question 1. Write the pseudo-code and implement an algorithm that takes as input a sensor network $(\mathcal{N}, R, c, \mathcal{A})$ with unit lifetimes and outputs M node-disjoint paths from s to t.

Question 2. What is the complexity of your algorithm?

3.2. Scheduling.

A scheduling of the network is a set of time intervals $[t_0, t_1]$, $[t_1, t_2]$,...,$[t_{n-1}, t_n]$, where $t_0 = 0$ and t_n is the network lifetime, and a set of scheduling functions $h_u(t) \in \{0, 1\}$ for every sensor $u \in \mathcal{N}$ constant on every interval

$[t_i, t_{i+1}]$. Fig. 3 gives an example of scheduling of four paths providing a $K = 2$ barrier coverage. We want to schedule the sensors so that the sensor network lifetime is maximized.

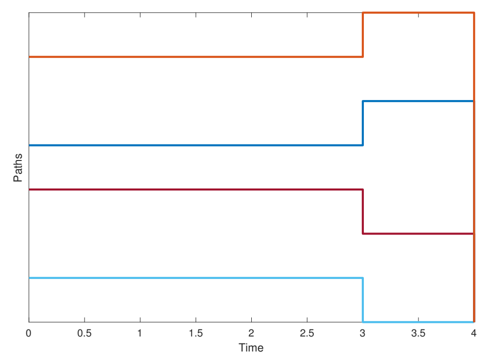

FIGURE 3. Example of scheduling of four paths providing a $K = 2$ barrier coverage (all sensors of a path have the same schedule), network lifetime is 4.

Question 3. Given M and K, propose a scheduling of the sensors that ensures the K-barrier coverage and maximizes the network lifetime.

3.3. **Numerical Application.** The file "sensornetwork0.txt" provides an example of sensor network with homogeneous lifetimes. Its format is the following: n (number of sensors), K, X_{max}, Y_{max}, R, all x-coordinates of the n sensors, all y-coordinates of the n sensors, c for every sensor.

Question 4. Give for this network the list of node-disjoint paths in Γ, the network lifetime and show the schedule in a convenient way. Provide the CPU time for solving the problem.

4. Heterogeneous Lifetimes

In this section, we assume that nodes may have integral heterogeneous lifetimes ($\forall u \in \mathcal{N}$, $c(u) \in \mathbb{N}^*$). We first determine the maximum K-route flow, then decompose it into elementary K-flows and at last compute the sensor scheduling.

4.1. **Maximum K-Route Flow.** Let $F(p)$ be the maximum flow value from s to t in the network $G^p = (V, E, w^p)$ where all edges capacities in G are replaced by $w^p(u, v) = \min\{p, w(u, v)\}$. Figure 2 (b) shows G^p for $p = 4$ for the graph G shown in (a). Here are some results, for which we omit the proof.

Theorem 1. *Let $\Psi(p) = F(p) - Kp$. Then, the maximum value of a K-route flow is $M \triangleq \max_{p \geq 0}\{F(p) : \Psi(p) \geq 0\}$.*

Lemma 3. *(i) The function $\Psi(p)$ is concave and piecewise linear. (ii) Each breakpoint is a multiple of $1/r$ for some integer $r \in \{1, ..., |E|\}$. The distance between two successive breakpoints in $\Psi(p)$ is at least $1/|E|^2$.*

Question 5. Using these results, write the pseudo-code an implement an algorithm that takes as inputs a directed graph $G = (V, E, w)$ and an integer K and outputs a maximum K-route flow and its value.

Question 6. What is the complexity of your algorithm?

4.2. **Decomposition of a K-Route Flow.** We now look for the decomposition of the K-route flow into elementary K-flows.

Question 7. For a directed graph $G = (V, E, w)$ with edge demands d, implement an algorithm that outputs a feasible flow.

Question 8. Propose a modification of the residual network and of the Edmond-Karp algorithm that starts from a feasible flow and find a maximum s-t flow in a directed graph $G = (V, E, w)$ with edge demands d.

Lemma 4. *Suppose that f is a feasible flow from s to t in $G = (V, E, w)$ with value $K\nu$ and satisfying $0 \leq f(u, v) \leq \nu$, $\forall u, v \in V$ and $K \in \mathbb{N}$. Then, there exists an elementary K-flow g such that $g(u, v) = 0$ if $f(u, v) = 0$ and $g(u, v) = 1$ if $f(u, v) = \nu$.*

Proof. Consider the maximum flow problem obtained by replacing the capacity on edge (u, v) by $w'(u, v) = \lceil f(u, v)/\nu \rceil$ and imposing a lower bound (or edge demand) of $l'(u, v) = \lfloor f(u, v)/\nu \rfloor$. Then, the flow f/ν is feasible for the maximum flow problem and has a flow of K units. The flow f/ν may not be integral; however by the integrality of the capacities and demands, there exists an integral flow g of the same value. Since the flow to the sink is K units, and the flow in each arc is 0 or 1, it follows that g is an elementary K-flow. Moreover, $g(u, v) = l'(u, v) = w'(u, v) = 0$ when $f(u, v) = 0$ and $g(u, v) = l'(u, v) = w'(u, v) = 1$ when $f(u, v) = \nu$. □

Lemma 5. *Suppose that f is a feasible flow in G from s to t, and the flow into the sink t is $K\nu$ for some integer K. Suppose further that $0 \leq f(u, v) \leq \nu$, $\forall u, v \in V$. Then, f is a K-route flow.*

Proof. If $\nu = 0$ the result is trivially true. Assume now that $\nu > 0$. We say that the edge (u, v) is *intermediate* if $0 < f(u, v) < \nu$. We prove the result by induction on the number of intermediate edges. If this number is 0, then select a flow g as in Lemma 4. We have $g(u, v) = 1$ whenever $f(u, v) = \nu$, so that $f = \nu \cdot g$ and the lemma is true. Assume now that the number of intermediate edges is $k > 0$ and that the lemma is true for $k - 1$ or less intermediate edges. Let g be the elementary K-flow as defined in Lemma 4. Let $\Delta_1 = \min\{\nu - f(u, v) : g(u, v) = 0\}$, $\Delta_2 = \min\{f(u, v) : g(u, v) = 1\}$, and $\Delta = \min\{\Delta_1, \Delta_2\}$. Let $f' = f - \Delta g$ and $\nu' = \nu - \Delta$. By Lemma 4, $\Delta > 0$ and $0 \leq f'(u, v) \leq \nu'$ for all edges $(u, v) \in E$. Moreover, the flow into the sink is $K(\nu - \Delta) = K\nu'$ and so f' and ν' satisfy the assumptions of Lemma 5. We now claim that the number of intermediate edges in f' is strictly less than in f. To see this, note that:

- If $f(u, v) = \nu$, then $f'(u, v) = \nu - \Delta = \nu'$.
- If $f(u, v) = 0$, then $f'(u, v) = 0$.

We conclude that each edge that is not intermediate with f is also not intermediate with f'. If $\Delta = \Delta_1$ and $\Delta_1 = \nu - f(u, v)$ for some $(u, v) \in E$. In this case, $f'(u, v) = f(u, v) = \nu - \Delta = \nu'$ and (u, v) is no longer intermediate. If $\Delta = \Delta_2$ and $\Delta_2 = f(u, v)$ for some $(u, v) \in E$. In this case, $f'(u, v) = 0$ and (u, v) is no longer intermediate. □

Question 9. Using Lemmas 4 and 5, write the pseudo-code and implement an algorithm that takes as inputs a directed graph $G = (V, E, w)$, an integer K and a K-route flow f, and outputs its decomposition in elementary K-flows with their respective value.

Question 10. What is the complexity of your algorithm?

4.3. **Scheduling.**

Question 11. Write the pseudo-code of an algorithm that takes as inputs an instance of the K barrier coverage problem $(\mathcal{N}, R, c, K, \mathcal{A})$ and outputs the set of node-disjoint paths with their respective scheduling and the network lifetime.

4.4. **Numerical Application.** The file "sensornetwork1.txt" provides an example of sensor network with heterogeneous lifetimes. Its format is the following: n (number of sensors), K, X_{max}, Y_{max}, R, all x-coordinates of the n sensors, all y-coordinates of the n sensors, c for every sensor.

Question 12. Give for this network the list of node-disjoint paths in Γ, their value, the network lifetime and show the schedule in a convenient way. Provide the CPU time for solving the problem.