Distributed Learning in Games for Wireless Networks

Ph.D. Defense

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NETLEARN Project

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Motivation

Figure: 5G Network requirements

Some Key Network Technologies

1. Ultra dense small cell networks
2. Device-to-device networks
3. Sensor networks

Some Classical Questions

1. Resource allocation?
2. Load balancing?
3. Coverage maximization?

Solution Approaches

1. Centralized approaches
2. Distributed learning

Distributed Learning in Games for Wireless Networks

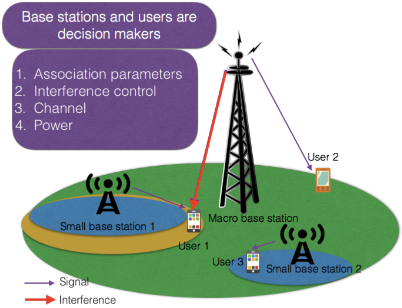


Figure: Small cell networks

- *Game:* Interactions can be modeled as a game
  - Players: Base stations and users
  - Strategies: Parameters
  - Cost/Utility: Related to data rate, load, ..
- *Nash Equilibrium (NE):* No player gains by deviating
- *Distributed Learning:* Players learn from their actions to reach the optimal NE

Outline of Presentation

1. Potential Games (PG)
2. Distributed learning in near-potential game
3. Distributed learning in noisy-potential game
4. Application 1: Load balancing in small cell networks
5. Application 2: Channel assignment in D2D networks

Game and Nash Equilibrium

Definition (Game)

A finite game $\mathcal{G} = \left\{\mathcal{S}, \{X_i\}_{i\in\mathcal{S}}, \{U_i\}_{i\in\mathcal{S}}\right\}$, where $\mathcal{S}$ is a set of players, $X = X_1 \times X_2 \times \ldots \times X_{|\mathcal{S}|}$ is action sets and $U_i : X \to \mathcal{R}$ is a cost or (utility) function. An action profile is denoted as $x = (x_i, x_{-i})$.

Definition ($\epsilon-$Nash Equilibrium)

An action profile $\left(x_i^*, x_{-i}^*\right) \in X$ is an ϵ-NE if

$$U_i(x_i^*, x_{-i}^*) - U_i(x_i, x_{-i}^*) \leq \epsilon, \ \forall i \in \mathcal{S}, x_i \in X_i. \tag{1}$$

If $\epsilon = 0$ then it is a Pure NE (PNE).

Potential Games

Definition (Potential games)

Consider a $\mathcal{G}$ if there is a potential function $\Phi : X \to \mathcal{R}$ such that $\forall i \in \mathcal{S}$, $\forall x_i, x_i' \in X_i$ and $\forall x_{-i} \in X_{-i}$ the below condition is true then the game is *exact-PG* if

$$U_i(x_i, x_{-i}) - U_i(x_i', x_{-i}) = \Phi(x_i, x_{-i}) - \Phi(x_i', x_{-i}), \tag{2}$$

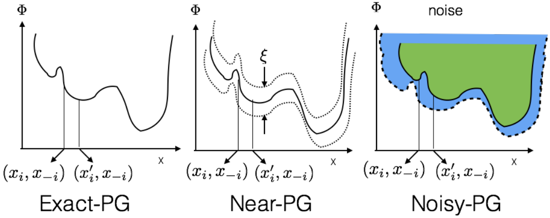
the game is *near-PG* if

$$\left| U_i(x_i, x_{-i}) - U_i(x_i', x_{-i}) + \Phi(x_i', x_{-i}) - \Phi(x_i, x_{-i}) \right| \leq \xi, \tag{3}$$

the game is *noisy-PG* if

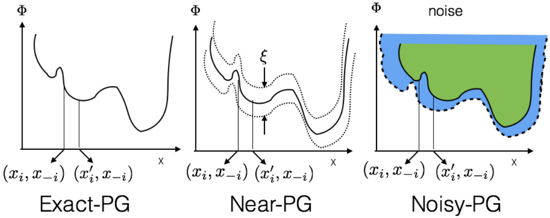
$$\mathbb{E}[U_i(x_i, x_{-i})] - \mathbb{E}\left[U_i(x_i', x_{-i})\right] = \Phi(x_i, x_{-i}) - \Phi(x_i', x_{-i}). \tag{4}$$

Potential Games



- Potential games are generalization of common payoff games
- Potential games have at least one NE [Monderer and Shapley, '96]
- All optimal points of potential function are NEs [Monderer and Shapley, '96]
- We call the global optimizer of potential function as the optimal NE.
- Log-linear learning algorithm (LLA) in exact-PG is known to converge to the optimal NE.

Related Work in Potential Games



- (Exact-PG) Log-linear learning algorithm (LLA) [Marden and Shamma, 2012]
- (Near-PG) LLA [Cannodagon, 2011]
- (Noisy-PG) Binary-LLA [Leslie and Marden, 2011]
- We extend the results of LLA and Binary-LLA in near-PG and noisy-PG
- We apply the obtained results to wireless network problems

Binary Log-linear Learning Algorithm (BLLA)

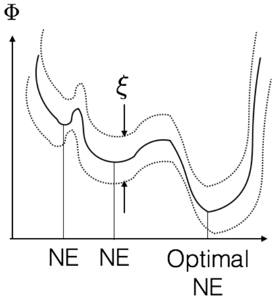
BLLA in near-PG

Initialisation: Start with arbitrary action profile x.
Set parameter τ.
while $t \geq 1$ **do**
    Select a player i and a trial action $\hat{x}_i \in X_i$ with uniform probability.
    Player i plays action $x_i(t-1)$ to compute the cost $U_i\left(x(t-1)\right)$.
    Player i plays the trial action $\hat{x}_i$ to compute its cost $U_i\left(\hat{x}_i, x_{-i}(t-1)\right)$.
    Player i selects action $x_i(t) \in \left(x_i(t-1), \hat{x}_i\right)$ with probability

$$\left(1 + e^{\Delta_i/\tau}\right)^{-1}, \tag{5}$$

$$\Delta_i = U_i\left(\hat{x}_i, x_{-i}(t-1)\right) - U_i\left(x(t-1)\right).$$

    All the other players repeat their actions i.e., $x_{-i}(t) = x_{-i}(t-1)$.
end while



- τ controls perturbation
    - $\tau \to 0$ BLLA $\equiv$ Better response
    - $\tau \to \infty$ BLLA $\equiv$ Uniform random
- For every $\tau > 0$, the underlying Markov chain is ergodic.
- Stochastically Stable States (SSSs) are the support of stationary distribution as $\tau \to 0$.

Rules for Resistance Computation

- Roots of the minimum resistance tree are SSSs.
- Resistance is the cost of transition of BLLA.
- We propose resistance rules that enables the convergence analysis of BLLA.

Proposition (Rules for Resistance Computation)

Let f, f_1 and f_2 be strictly positive functions. Let resistances $Res(f)$, $Res(f_1)$, and $Res(f_2)$ exist. Let κ be a positive constant.

Rule 1. $f_1(\tau)$ *is sub-exponential if and only if* $Res(f_1) = 0$. *In particular* $Res(\kappa) = 0$.

Rule 2. $Res(e^{-\kappa/\tau}) = \kappa$.

Rule 3. $Res(f_1 + f_2) = \min\{Res(f_1), Res(f_2)\}$.

Rule 4. *If* $Res(f_1) < Res(f_2)$ *then* $Res(f_1 - f_2) = Res(f_1)$.

Rule 5. $Res(f_1 f_2) = Res(f_1) + Res(f_2)$.

Rule 6. $Res(\frac{1}{f}) = -Res(f)$.

Rule 7. *If* $\forall\tau, f_1(\tau) \leq f_2(\tau)$ *then* $Res(f_2) \leq Res(f_1)$.

Rule 8. *If* $\forall\tau, f_1(\tau) \leq f(\tau) \leq f_2(\tau)$ *and if* $Res(f_1) = Res(f_2)$ *then* $Res(f)$ *exists and* $Res(f) = Res(f_1)$.

Convergence of BLLA in Near-PG

Let Φ^* and $\Phi^\dagger$ be the first minimum and second minimum values of the function Φ. Let $|X|$ be the cardinality of action space.

Theorem (Convergence to Optimal ϵ-NE)

For any ξ-potential game $\mathcal{G}$ with potential Φ and $\epsilon > 0$ the stochastically stable states of LLA and BLLA corresponds to a set of ϵ-NEs with potential less than $\Phi^* + \epsilon$ if

$$\xi < \frac{\epsilon}{2\left(|X| - 1\right)}. \tag{6}$$

Corollary (Convergence to Optimal PNE)

The stochastically stable states of LLA and BLLA corresponds to a set of PNEs whose potential is Φ^* if

$$\xi < \frac{\Phi^\dagger - \Phi^*}{2(|X| - 1)}. \tag{7}$$

- If the distance to exact PG is small then BLLA converges to optimal ϵ-NE.
- If the distance is sufficiently small then BLLA converges to optimal PNE.

BLLA in Noisy-PG

- (Key Idea) Use multiple samples of the noisy utilities.
- When a specific number of samples are used then BLLA converges.

Theorem

For any noisy potential game $\hat{\mathcal{G}}^N$ with potential Φ the stochastically stable states of BLLA are the global maximizers of the potential Φ if one of the following holds.

1. *The estimation noise is bounded in an interval of size ℓ and the number of estimation samples used are*
$$N \geq \left(\log\left(\frac{4}{\xi}\right) + \frac{2}{\tau}\right) \frac{\ell^2}{2\left(1-\xi\right)^2 \tau^2}, \tag{8}$$
where $0 < \xi < 1$.
2. *The estimation noise is unbounded with finite mean and variance. Let $M(\theta)$ be a finite moment generating function of noise. Let $\theta^* = \mathrm{argmax}_\theta\, \theta\left(1-\xi\right)\tau - \log\left(M(\theta)\right)$. The number of samples used are*
$$N \geq \frac{\log\left(\frac{4}{\xi}\right) + \frac{2}{\tau}}{\log\left(\frac{e^{\theta^*(1-\xi)\tau}}{M(\theta^*)}\right)}. \tag{9}$$

BLLA in Noisy-PG

Theorem (Almost sure convergence)

Consider BLLA in a noisy-PG with a decreasing parameter $\tau(t) = 1/\log(1+t)$*, and the number of samples* $N(\tau)$ *is given by above Theorem. Then, BLLA converges with probability* 1 *to the global maximizer of the potential function.*

- As τ decreases the number of samples N increases.
- In applications, a small N is desired.

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Load balancing in small cell networks

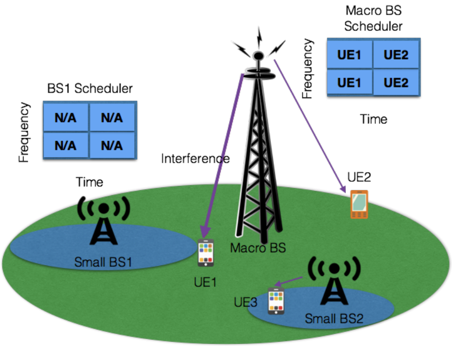

Figure: Load balancing in small cell network

- Association is based on maximum received power $P_i g_i(x)$

Load balancing in small cell networks

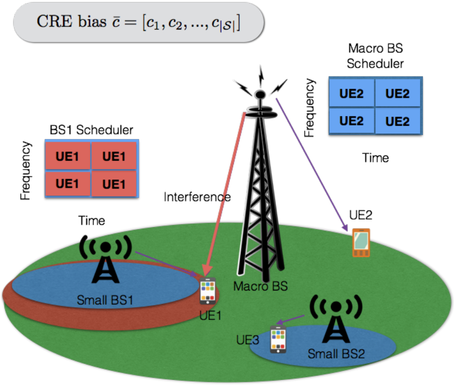

Figure: Load balancing in small cell network

- Cell Range Extension (CRE) bias increases the coverage range.
- With CRE association is based on maximum biased received power $P_i g_i(x) c_i$.
- (Outage) High interference may lead to user being not served.

Load balancing in small cell networks

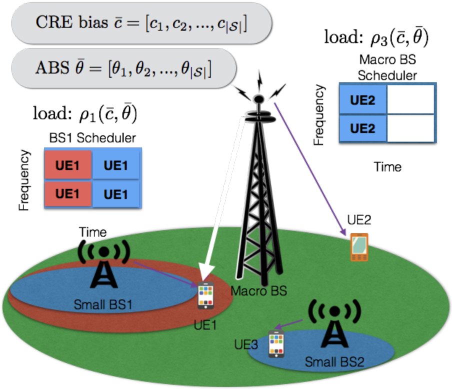

Figure: Load balancing in small cell network

- Cell Range Extension (CRE) bias increases the coverage range.
- (Outage) High interference may lead to user being not served.
- Almost Blank Subframe (ABS) helps reduce outages.
- θ_i is the proportion of blank subframes.

Load balancing in small cell networks

Global Objective Function

$$\phi_\alpha\left(\bar{c},\bar{\theta}\right)=\sum_{i\in\mathcal{S}} f_\alpha\left(\rho_i\left(\bar{c},\bar{\theta}\right)\right) \tag{10}$$

Minimization Problem

$$\left\{\bar{c}^*,\bar{\theta}^*\right\}=arg\ min_{\mathcal{F}}\phi_\alpha\left(\bar{c},\bar{\theta}\right) \tag{11}$$

- $(\alpha=0)$: Rate optimal policy
- $(\alpha=1)$: Proportional fair policy
- $(\alpha=2)$: Delay optimal policy
- $(\alpha\to\infty)$: Min-max load policy

Related Work:

- Convex optimization [Kim et al. 2012]
- Integer programming [Ye et al. 2013]
- Other work includes simulations, heuristics, sub-optimal algorithms

Load balancing in small cell networks

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Game Model

The interactions of BSs is modeled as a game $\Gamma=\left\{\mathcal{S},\{X_i\}_{i\in\mathcal{S}},\{U_i\}_{i\in\mathcal{S}}\right\}$, where $\mathcal{S}$ is a set of BSs, X_i is strategy set, and U_i are cost functions:

$$U_i\left(a_i,a_{-i}\right)=\sum_{j\in N_i^{\varpi}} f_\alpha\left(\rho_j\left(a_i,a_{-i}\right)\right). \qquad (12)$$

- N_i^{ϖ} is a neighborhood of BS i, and ϖ is neighborhood control parameter:

$$\varpi=0 \implies N_i^0=\mathcal{S}$$
$$\varpi=1 \implies N_i^1=BS\ i$$
$$0<\varpi<1 \implies N_i^{\varpi}\subset\mathcal{S}$$

- The game Γ is a ξ^{ϖ}-PG
- We adapt BLLA for near-PG Γ

Neighborhood Scenario

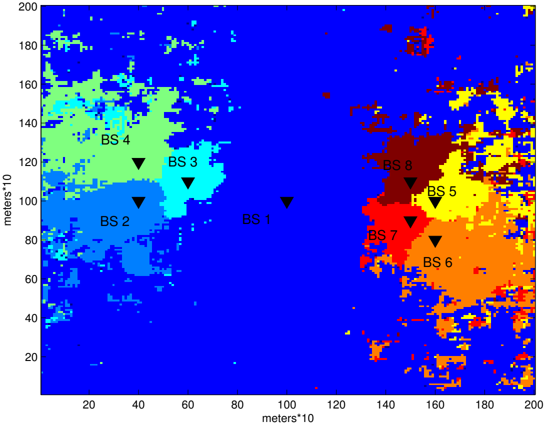


Figure: Large neighborhood with pathloss + fading

- All BSs can be neighbours with non-zero probability because of fading.
- Leads to centralised alogorithm.
- Distributed algorithm is obtained by limiting neighbors.
- Near-PG is best suitable in this scenario.

Results

Theorem (Sufficient Condition for Optimal ϵ-NE)

The constraint $\xi^{\varpi} < \frac{\epsilon}{2(|X|-1)}$ *is satisfied if*

$$\varpi \leq \epsilon M \left(1 - \rho_{\max}\right)^{\alpha},$$

where $\rho_{\max}$ *is the maximum load of a BS and* M *is a constant that depends on the system parameters.*

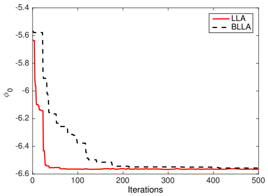
Corollary (Sufficient Condition for Optimal PNE)

The constraint $\xi^{\varpi} < \frac{\phi_{\alpha}^{\dagger} - \phi_{\alpha}^{*}}{2|X|}$ *is satisfied if*

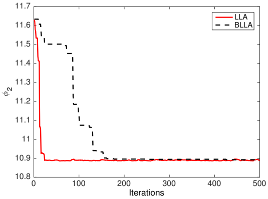
$$\varpi \leq M \left(\phi_{\alpha}^{\dagger} - \phi_{\alpha}^{*}\right) \left(1 - \rho_{\max}\right)^{\alpha}.$$

- As ϖ decreases the neighborhood expands that decreases the distance to exact-PG.
- A smaller neighborhood is desired so as to limit the information exchange.
- There is a tradeoff between the size of neighborhood and the quality of solution obtained.

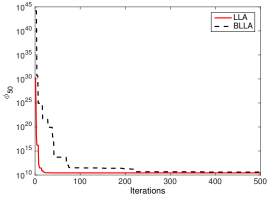
Convergence of BLLA



(a) ($\alpha = 0$) Rate-optimal.



(b) ($\alpha = 2$)Delay-optimal.

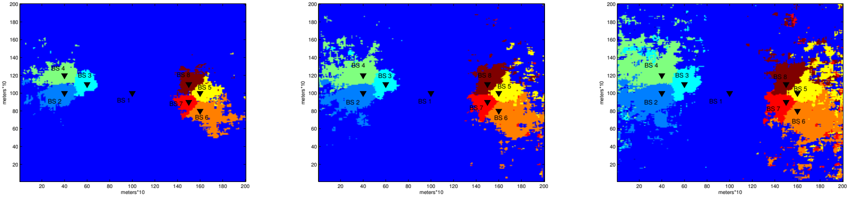


(c) ($\alpha = 50$) Min-max.

Figure: Convergence of BLLA ($\tau = 0.001$, $\varpi = 10^{-22}$, one macro BS, and 7 small BSs, maximum bias is 16).

	$\alpha = 0$		$\alpha = 2$		$\alpha \to \infty$	
BS i	c_i^*	ρ_i^*%	c_i^*	ρ_i^*%	c_i^*	ρ_i^*%
MBS 1	1	**92**	1	**61**	1	**45**
SBS 2	1	7	3	20	8	42
SBS 3	1	4	3	9	9	23
SBS 4	1	9	3	18	8	37
SBS 5	1	11	3	21	7	37
SBS 6	1	8	3	20	7	43
SBS 7	1	5	3	11	8	30
SBS 8	1	7	3	19	6	37

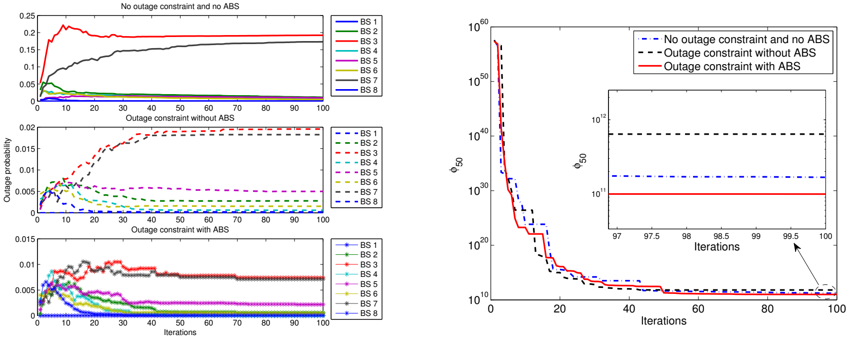
Optimal Coverage Regions



(a) ($\alpha = 0$) Rate-optimal. **(b)** ($\alpha = 2$)Delay-optimal. **(c)** ($\alpha = 50$) Min-max.

- In this setting, optimal coverage regions of small BSs grows with α as the optimal CRE bias increases.

Effect of ABS



(a) Outage constraint **(b)** Global cost

Figure: Improvement with ABS

- Outage constraint is met by restricting actions.
- ABS provides a new flexibility in obtaining a better load balancing with lower global cost.

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Channel Assignment Problem (CAP) in D2D Networks

D2D Network Model:

- Consider $\mathcal{D}$ set of all users
- $\mathcal{F}$ set of channels
- $\bar{c} = \left[c_1, c_2, \dots, c_{|\mathcal{D}|}\right]$ is channel vector
- $\hat{\nu}_j(\bar{c})$ is noisy estimated data rate
- Average sum data rate
$\phi(\bar{c}) = \mathbb{E}\left[\sum_{j \in \mathcal{D}} \hat{\nu}_j(\bar{c})\right]$

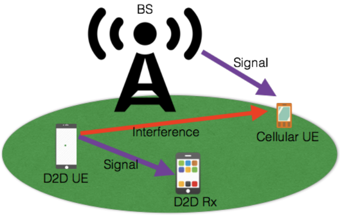


Figure: D2D network model showing signal and interference links.

Goal (Maximize expected sum data rate)

$$\bar{c}^* \in \underset{\bar{c} \in \mathcal{F}^{|\mathcal{D}|}}{\text{argmax}}\ \phi(\bar{c})$$

- Dynamic programming [Wang, 2016], graph-theoretical and heuristic solutions [maghsudi, 2016], game theory [song, 2014], etc..
- Even in centralized, full information, no-noise case this is NP-hard

Noisy-potential Game Model

Definition (CAP Game)

A CAP game $\hat{\mathcal{G}} := \{\mathcal{D}, \{X_i\}_{i\in\mathcal{D}}, \{\hat{U}_i\}_{i\in\mathcal{D}}\}$*, where* $\mathcal{D}$ *is a set of UEs that are players of the game,* $\{X_i\}_{i\in\mathcal{D}}$ *are action sets consisting of orthogonal channels,* $\hat{U}_i : X \to \mathcal{R}$ *are random utility functions, and* $X := X_1 \times X_2 \times \ldots X_{|\mathcal{D}|}$.

Proposition

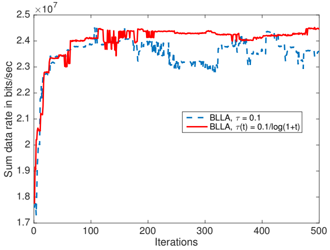
CAP game $\hat{\mathcal{G}}^{\mathcal{N}} := \{\mathcal{D}, \{X_i\}_{i\in\mathcal{D}}, \{\hat{U}_i^N\}_{i\in\mathcal{D}}\}$ *is a noisy-potential game with potential* $\phi(a)$ *if* $\hat{U}_i^N = \frac{1}{N}\sum_{k=1}^{N} \hat{U}_i$*, and marginal contribution utility*

$$\hat{U}_i(a_i, a_{-i}) = \sum_{j\in\mathcal{D}(a_i)} \hat{\nu}_j(a_i, a_{-i}) - \sum_{j\in\mathcal{D}(a_i)\setminus i} \hat{\nu}_j(a_i^0, a_{-i}),$$

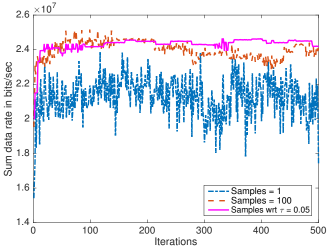
where $\mathcal{D}(a_i)$ *is the set of UEs on channel* a_i*, and* a_i^0 *is a null action.*

- We apply BLLA for the noisy-PG $\hat{\mathcal{G}}^{\mathcal{N}}$ by using the number of samples N is given in Theorem.

Convergence of BLLA in noisy-PG



(a) Convergence of BLLA for fixed temperature and decreasing temperature.



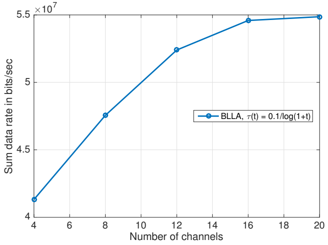
(b) Effect of number of samples on convergence of BLLA.

(a) Decreasing temperature results in smooth convergence compared to fixed

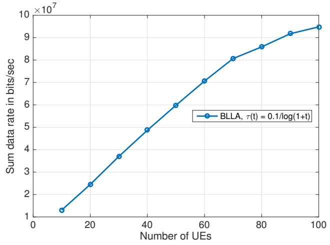
(b) For $\tau = 0.05$, the number of samples N from the Theorem gives smooth convergence. Otherwise, high fluctuations are observed.

(c) No guarantee of convergence for other number of samples.

Sum Data Rate Vs Number of Channels and UEs



(a) Effect of channels on sum rate by fixing #UEs to 20.



(b) Effect of number of UEs on sum rate by fixing #channels to 10.

(a) Sum rate increases with the number of channels since BLLA assign channels optimally leading to lower interference per channel

(b) Sum rate increases linearly until 60 UEs as BLLA manages to assign channels optimally and maintain low interference.

Conclusion and Future Works

Conclusion

- Extended the results to broader class of potential games: near-PG and noisy-PG
- Provided new rules for proving convergence
- Proposed an algorithm to learn the best parameter τ of LLA
- Load balancing in small cell network
- Channel assignment in D2D network
- Characterize the efficiency of distributed greedy algorithm for submodular maximization

Future Work

- Relationship of near-PG and noisy-PG
- Analysis of greedy algorithm with other utility functions
- Application to sensor networks

Publications I

Journal Papers

J1 Mohd. Shabbir Ali, Pierre Coucheney, and Marceau Coupechoux "*Load Balancing in Heterogeneous Networks Based on Distributed Learning in Near-Potential Games*", *IEEE Trans. Wireless Commun.*, vol.15, no.7, pp.5046-5059, 2016.

J2 Mohd. Shabbir Ali, Pierre Coucheney, and Marceau Coupechoux "*Optimal distributed channel assignment in D2D networks using learning in noisy potential games*", To be submitted *IEEE Trans. Wireless Commun.*, 2017.

J3 Mohd. Shabbir Ali, David Grimsman, João P. Hespanha and Jason R. Marden " *Efficiency and Information Trade-off of Submodular Maximization Problems*", To be Submitted *IEEE Transactions on Automatic Control*, 2017.

J4 Mohd. Shabbir Ali and Neelesh B. Mehta, "Modeling Time-Varying Aggregate Interference in Cognitive Radio Systems, and Application to Primary Exclusive Zone Design," *IEEE Trans. Wireless Commun.*, vol.13, no.1, pp.429-439, Jan. 2014.

arXiv Paper

X1 Mohd. Shabbir Ali, Pierre Coucheney, and Marceau Coupechoux " *Optimal distributed channel assignment in D2D networks using learning in noisy potential games*", *arXiv preprint arXiv:1701.04577*, 2017.

Publications II

Conference Papers

C1 Mohd. Shabbir Ali, Pierre Coucheney, and Marceau Coupechoux "*Load Balancing in Heterogeneous Networks Based on Distributed Learning in Potential Games*", Proc. *IEEE WiOpt,* pp.371-378, May 2015.

C2 Mohd. Shabbir Ali, Pierre Coucheney, and Marceau Coupechoux " *Optimal distributed channel assignment in D2D networks using learning in noisy potential games*", *Accepted in INFOCOM 5G and Beyond Workshop*, May 2017.

C3 Mohd. Shabbir Ali and N. B. Mehta, "*Modeling time-varying aggregate interference from cognitive radios and implications on primary exclusive zone design*", *IEEE Global Communications Conference (GLOBECOM)*, pp. 3760-3765, 2013.

C4 Mohd. Shabbir Ali, Pierre Coucheney, and Marceau Coupechoux "*Learning Annealing Schedule of Log-Linear Algorithms for Load Balancing in HetNets*", *Proc. European Wireless Conference* pp. 1-6, 2016.

C5 Mohd. Shabbir Ali, Pierre Coucheney, and Marceau Coupechoux " *Rules for Computing Resistance of Transitions of Learning Algorithms in Games*", Accepted in *International Conference on Game Theory for Networks (GAMENETS)*, May 2017.

C6 Mohd. Shabbir Ali, David Grimsman, João P. Hespanha and Jason R. Marden " *Efficiency and Information Trade-off of Submodular Maximization Problems*", In Review *IEEE Conference on Decision and Control (CDC)*, 2017.

THANK YOU!