A Fluid Model for Performance Analysis in Cellular Networks

Jean-Marc Kelif [a,*],

[a] *France Telecom R&D, Issy-Les-Moulineaux, France*

Marceau Coupechoux [b], Philippe Godlewski [b]

[b] *TELECOM ParisTech & CNRS LTCI UMR 5141, Paris, France*

Abstract

In this paper, we propose a new framework to study the performance of cellular networks using a fluid model and we derive from this model analytical formulas for interference, outage probability, and spatial outage probability. The key idea of the fluid model is to consider the discrete base stations (BS) entities as a continuum of transmitters that are spatially distributed in the network. This model allows us to obtain simple analytical expressions to reveal main characteristics of the network. In this paper, we focus on the downlink other-cell interference factor (OCIF), which is defined for a given user as the ratio of its outer cell received power to its inner cell received power. A closed-form formula of the OCIF is provided in this paper. From this formula, we are able to obtain the global outage probability as well as the spatial outage probability, which depends on the location of a mobile station (MS) initiating a new call. Our analytical results are compared to Monte Carlo simulations performed in a traditional hexagonal network. Furthermore, we demonstrate an application of the outage probability related to cell breathing and densification of cellular networks.

Key words: cellular networks, interference modeling, interference factor, OCIF, fluid model, outage probability, cell breathing, network densification.

* Corresponding author. Address: 38-40, rue du Général Leclerc, 92794 Issy-Les-Moulineaux, France. Tel: +33 1 45 29 61 79. Fax: +33 1 45 29 41 94.

Email addresses: jeanmarc.kelif@orange-ftgroup.com (Jean-Marc Kelif), coupecho@enst.fr (Marceau Coupechoux), godlewsk@enst.fr (Philippe Godlewski).

1 Introduction

Estimation of cellular networks capacity is one of the key points before deployment and mainly depends on the characterization of interference. As downlink is often the limited link w.r.t. capacity, we focus on this direction throughout this paper, although the proposed framework can easily be extended to the uplink. An important system parameter for this characterization is the other-cell interference factor (OCIF). It represents the 'weight' of the network on a given cell.

OCIF is traditionally defined as the ratio of other-cell interference to inner-cell interference. In this paper, we rather consider an OCIF, which is defined as the ratio of total other-cell received power to the total inner-cell received power. Although very close, this new definition is interesting for three reasons. Firstly, total received power is the metric mobile stations (MS) are really able to measure on the field. Secondly, the power ratio is now a characteristic of the network and does not depend on the considered MS or service. At last, the definition of OCIF is still valid if we consider cellular systems without inner-cell interference; in this case, the denominator of the ratio is reduced to the useful power. The precise knowledge of the OCIF allows the derivation of performance parameters, such as outage probabilities, capacity, as well as the definition of Call Admission Control mechanisms, in CDMA (Code Division Multiple Access) and OFDMA (Orthogonal Frequency Division Multiple Access) systems.

Pioneering works on the subject [1] mainly focused on the uplink. Working on this link, [5] derived the distribution function of a ratio of path-losses, which is essential for evaluating the external interference. For this purpose, authors approximated the hexagonal cell with a disk of same area. Based on this result, Liu and Everitt proposed in [6] an iterative algorithm for the computation of the OCIF for the uplink.

On the downlink, [2] [3] aimed at computing an averaged OCIF over the cell by numerical integration in hexagonal networks. In [8], Gilhousen et al. provided Monte Carlo simulations and obtained a histogram of the OCIF. In [7], other-cell interference was given as a function of the distance to the base station (BS) using Monte-Carlo simulations. Chan and Hanly [9] precisely approximated the distribution of the other-cell interference. They however provided formulas that are difficult to handle in practice. Baccelli et al. [15] studied spatial blocking probabilities in random networks. Focusing on random networks, power ratios were nevertheless not their main concern and authors relied on approximated formulas, which are not validated by simulations.

In contrast to previous works in the field, the modeling key of our approach is to consider the discrete BS entities of a cellular network as a continuum. Recently, the authors of [17] described a network in terms of macroscopic quantities such as node density. The same idea was used in [18] for ad hoc networks. They however assumed a very high density of nodes in both papers and infinite networks. We show hereafter that our model is accurate even when the density of BS is very low and the network size is limited (see section 3.2).

Central idea of this paper has been originally proposed in conference papers [10–13], which provide a simple closed-form formula for the OCIF on the downlink as a function of the distance to the BS, the path-loss exponent, the distance between two BS, and the network size. We validate here the formulas by Monte Carlo simulations and show that it is possible to get a simple outage probability approximation by integrating the OCIF over a circular cell. In addition, as this ratio is obtained as a function of the distance to the BS, it is possible to derive a spatial outage probability, which depends on the location of a newly initiated call. Outage probability formula allows us to analyze the phenomenon of cell breathing, which results in coverage holes when the traffic load increases. An answer to this issue is to increase the BS density (i.e., network densification). Thanks to the proposed formulas, we are able to quantify this increase.

We first introduce the interference model and the notations (section 2), then present the fluid model and its validation (section 3). In the fourth section, we derive outage probabilities. In the last section, we apply the theoretical results to the characterization of cell breathing and to network densification.

2 Interference Model and Notations

We consider a cellular network and we focus on the downlink. BS have omnidirectional antennas, so that a BS covers a single cell. Let us consider a mobile station u and its serving base station b.

The propagation path gain $g_{j,u}$ designates the inverse of the path-loss pl between base station j and mobile u, i.e., $g_{j,u} = 1/pl_{j,u}$.

The following power quantities are considered:

- $P_{b,u}$ is the transmitted power from serving base station b to mobile u (for user's traffic);
- P_j is the total transmitted power by a generic base station j;
- $P_b = P_{cch} + \Sigma_u P_{b,u}$ is the total power transmitted by station b, P_{cch} represents the amount of power used to support broadcast and common control channels;

- $p_{b,u}$ is the power received at mobile u from station b; we can write $p_{b,u} = P_b \, g_{b,u}$;
- $S_u = S_{b,u} = P_{b,u} \, g_{b,u}$ is the useful power received at mobile u from serving station b (for traffic data). Since we do not consider soft-handover (SHO), serving station is well defined and the subscript b can be omitted for the sake of readability.

The total amount of power experienced by a mobile station u in a cellular system consists of three terms: useful signal power (S_u), interference and noise power (N_{th}). It is common to split the system power into two terms: $p_{int,u} + p_{ext,u}$, where $p_{int,u}$ is the *internal* (or own-cell) received power and $p_{ext,u}$ is the *external* (or other-cell) interference. Note that we made the choice of including the useful signal S_u in $p_{int,u}$, and, as a consequence, it should not be confused with the commonly considered own-cell interference.

With the above notations, we define the OCIF in u, as the ratio of total power received from other BS to the total power received from the serving BS b as:

$$f_u = \frac{p_{ext,u}}{p_{int,u}}. \tag{1}$$

The quantities f_u, $p_{ext,u}$, and $p_{int,u}$ are location dependent and can thus be defined in any location x as long as the serving BS is known.

In this paper, we use the signal to interference plus noise ratio (SINR) as the criteria of radio quality: γ_u^* is the SINR target for the service requested by MS u. This figure is a priori different from the SINR γ_u evaluated at mobile station u. However, we assume perfect power control, so that $SINR = \gamma_u^*$ for all users.

2.1 CDMA Network

On the downlink of CDMA networks, orthogonality between physical channels may be approached by Hadamard multiplexing if the delay spread is much smaller than the chip duration T_c. As a consequence, a coefficient α may be introduced to account for the lack of perfect orthogonality in the own cell.

With the introduced notations, the SINR experimented by u can thus be derived (see e.g. [4]):

$$\gamma_u^* = \frac{S_u}{\alpha(p_{int,u} - S_u) + p_{ext,u} + N_{th}}. \tag{2}$$

From this relation, we can express S_u as:

$$S_u = \frac{\gamma_u^*}{1+\alpha\gamma_u^*}\, p_{int,u}\, (\alpha + p_{ext,u}/p_{int,u} + N_{th}/p_{int,u}). \quad (3)$$

As we defined the OCIF as $f_u = p_{ext,u}/p_{int,u}$, we have:

$$f_u = \frac{\sum_{j\neq b} P_j g_{j,u}}{P_b g_{b,u}}. \quad (4)$$

In case of a homogeneous network, $P_j = P_b$ for all j and:

$$f_u = \frac{\sum_{j\neq b} g_{j,u}}{g_{b,u}}. \quad (5)$$

The transmitted power for MS u, $P_{b,u} = S_u/g_{b,u}$ can now be written as:

$$P_{b,u} = \frac{\gamma_u^*}{1+\alpha\gamma_u^*}(\alpha P_b + f_u P_b + N_{th}/g_{b,u}). \quad (6)$$

From this relation, the output power of BS b can be computed as follows:

$$P_b = P_{cch} + \sum_u P_{b,u}, \quad (7)$$

and so, according to Eq.6,

$$P_b = \frac{P_{cch} + \sum_u \frac{\gamma_u^*}{1+\alpha\gamma_u^*}\frac{N_{th}}{g_{b,u}}}{1 - \sum_u \frac{\gamma_u^*}{1+\alpha\gamma_u^*}(\alpha + f_u)}. \quad (8)$$

In HSDPA (High Speed Downlink Packet Access), Eq.2 and 8 are valid for each TTI (Transmission Time Interval). Parameter γ_u^* should be now interpreted as an experienced SINR and not any more as a target. Sums are done on the number of scheduled users per TTI. If a single user is scheduled per TTI (which is a common case), sums reduce to a single term. Even in this case, parameter α has to be considered since common control channels and traffic channels are not perfectly orthogonal due to multi-path.

Even if previous equations use the formalism of CDMA networks, it is worth noting that they are still valid for other multiplexing schemes. For cellular technologies without internal interference (TDMA, Time Division Multiple Access, OFDMA), $p_{int,u}$ reduces to S_u and $p_{ext,u}$ is the co-channel interference in Eq.2. The definition of f_u is unchanged provided that sums in Eq.4 and 5 are done over the set of co-channel interfering BS (to account for frequency reuses different from 1).

2.2 OFDMA Network

In OFDMA, Eq.2 can be applied to a single carrier. P_b is now the base station output power per subcarrier. In OFDMA, data is multiplexed over a great number of subcarriers. There is no internal interference, so we can consider that $\alpha(p_{int,u} - S_u) = 0$. Since $p_{ext,u} = \sum_{j \neq b} P_j g_{j,u}$, we can write:

$$\gamma_u = \frac{P_{b,u} g_{b,u}}{\sum_{j \neq b} P_j g_{j,u} + N_{th}}, \tag{9}$$

so we have:

$$\gamma_u = \frac{1}{f_u + \frac{N_{th}}{P_{b,u} g_{b,u}}}. \tag{10}$$

Moreover, when $\frac{N_{th}}{P_{b,u} g_{b,u}} << f_u$, which is typically verified for cell radii less than about 1 Km, we can neglect this term and write

$$\gamma_u = \frac{1}{f_u}. \tag{11}$$

For each subcarrier of an OFDMA system (e.g. WiMax, LTE), the parameter f_u represents the inverse of the SIR (Signal to Interference Ratio).

Consequently, f_u appears to be an important parameter characterizing cellular networks. This is the reason why we focus on this factor in the next section and, with the purpose of proposing a closed form formula of f_u, we develop a physical model of the network.

3 Fluid Model

In this section, we first present the model, derive the closed-form formula for f_u, and validate it through Monte-Carlo simulations for a homogeneous hexagonal network.

The key modelling step of the model we propose consists in replacing a given fixed finite number of interfering BS by an equivalent continuum of transmitters, which are spatially distributed in the network. This means that the transmitting interference power is now considered as a continuum field all over the network. In this context, the network is characterized by a MS density ρ_{MS} and a co-channel base station density ρ_{BS} [10]. We assume that MS and BS are uniformly distributed in the network, so that ρ_{MS} and ρ_{BS} are constant. As the network is homogeneous, all base stations have the same output power P_b.

We focus on a given cell and consider a round shaped network around this central cell with radius R_{nw}. In Figure 1, the central disk represents the cell of interest, i.e., the area covered by its BS. The continuum of interfering BS is located between the dashed circle and the outer circle. By analogy with the discrete regular network, where the half distance between two BS is R_c, we consider that the minimum distance to interferers is $2R_c$.

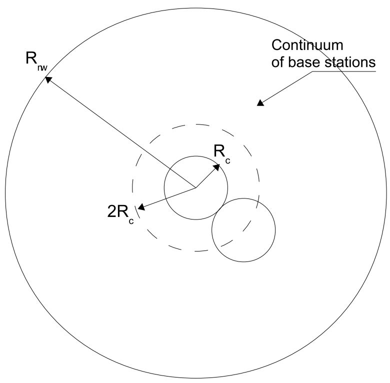

Fig. 1. Network and cell of interest in the fluid model; the minimum distance between the BS of interest and interferers is $2R_c$ and the interfering network is made of a continuum of base stations.

For the assumed omni-directional BS network, we use a propagation model, where the path gain, $g_{b,u}$, only depends on the distance r between the BS b and the MS u. The power, $p_{b,u}$, received by a mobile at distance r_u can thus be written $p_{b,u} = P_b K r_u^{-\eta}$, where K is a constant and $\eta > 2$ is the path-loss exponent.

Let us consider a mobile u at a distance r_u from its serving BS b. Each elementary surface $zdzd\theta$ at a distance z from u contains $\rho_{BS} zdzd\theta$ base stations which contribute to $p_{ext,u}$. Their contribution to the external interference is $\rho_{BS} zdzd\theta P_b K z^{-\eta}$. We approximate the integration surface by a ring with centre u, inner radius $2R_c - r_u$, and outer radius $R_{nw} - r_u$ (see Figure 2).

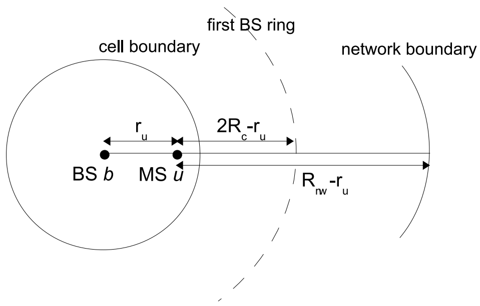

Fig. 2. Integration limits for external interference computation.

$$p_{ext,u} = \int_{0}^{2\pi} \int_{2R_c - r_u}^{R_{nw} - r_u} \rho_{BS} P_b K z^{-\eta} z dz d\theta$$

$$= \frac{2\pi \rho_{BS} P_b K}{\eta - 2} \left[(2R_c - r_u)^{2-\eta} - (R_{nw} - r_u)^{2-\eta} \right]. \quad (12)$$

Moreover, MS u receives internal power from b, which is at distance r_u: $p_{int,u} = P_b K r_u^{-\eta}$. So, the OCIF $f_u = p_{ext,u}/p_{int,u}$ can be expressed by:

$$f_u = \frac{2\pi \rho_{BS} r_u^{\eta}}{\eta - 2} \left[(2R_c - r_u)^{2-\eta} - (R_{nw} - r_u)^{2-\eta} \right]. \quad (13)$$

Note that f_u does not depend on the BS output power. This is due to the assumption of a homogeneous network (all base stations have the same transmit power). In our model, f_u only depends on the distance r_u to the BS. Thus, if the network is large, i.e., R_{nw} is large compared to R_c, f_u can be further approximated by:

$$f_u = \frac{2\pi \rho_{BS} r_u^{\eta}}{\eta - 2} (2R_c - r_u)^{2-\eta}. \quad (14)$$

This closed-form formula will allow us to quickly compute performance parameters of a cellular network. However, before going ahead, we need to validate the different approximations we made in this model.

3.2 Validation of the Fluid Model

In this section, we validate the fluid model presented in the last section. In this perspective, we will compare the figures obtained with Eq.13 to those obtained numerically by simulations. Our simulator assumes a homogeneous hexagonal network made of several rings surrounding a central cell. Figure 3 shows an example of such a network with the main parameters involved in the study: R_c, the half distance between BS, $R = 2\sqrt{3}R_c/3$, the maximum distance in the hexagon to the BS, and R_{nw}, the range of the network.

The fluid model and the traditional hexagonal model are both simplifications of the reality. None is a priori better than the other but the latter is widely used, especially for dimensioning purposes. That is the reason why a comparison is useful.

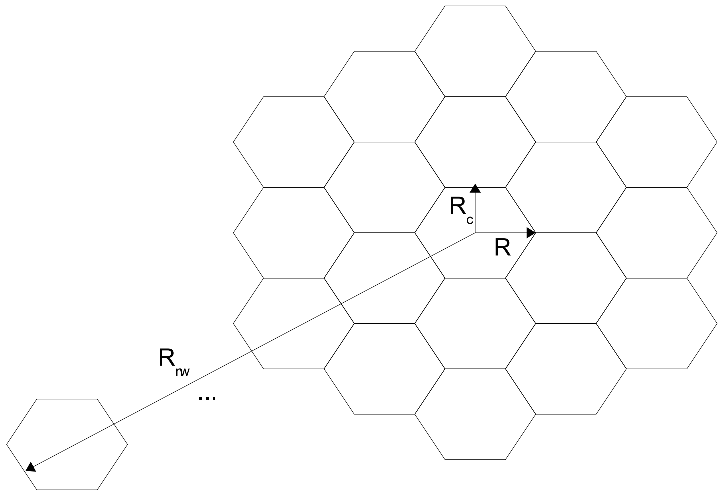

Fig. 3. Hexagonal network and main parameters of the study.

The validation is done by Monte Carlo simulations:

- at each snapshot, a location is randomly chosen for MS u in the cell of interest b with uniform spatial distribution;
- f_u is computed using Eq.5 with $g_{j,u} = Kr_{j,u}^{-\eta}$, where $r_{j,u}$ is the distance between the BS j and the MS u. The serving BS b is the closest BS to MS u;
- the value of f_u and the distance to the central BS b are recorded;
- at the end of the simulation, all values of f_u corresponding to a given distance are averaged and we plot the average value in Figure 4.

Figure 4 shows the simulated OCIF as a function of the distance to the base station. Simulation parameters are the following: $R = 1$ Km; η between 2.7 and 4; $\rho_{BS} = (3\sqrt{3}R^2/2)^{-1}$; R_{nw} is chosen, such that the number of rings of interfering BS is 15; and the number of snapshots is 1000. Eq.13 is also plotted for comparison. In all cases, the fluid model matches very well the simulations on a hexagonal network for various figures of the path-loss exponent. Note that at the border of the cell (between 0.95 and 1 Km), the model is a little bit less accurate because hexagon corners are not well captured by the fluid model.

Note that the considered network size can be finite and chosen to characterize each specific local network environment. Figure 5 shows the influence of the

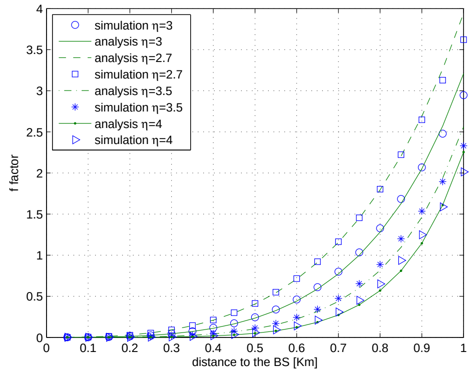

Fig. 4. OCIF vs. distance to the BS; comparison of the fluid model with simulations on a hexagonal network with $\eta = 2.7$, 3, 3.5, and 4.

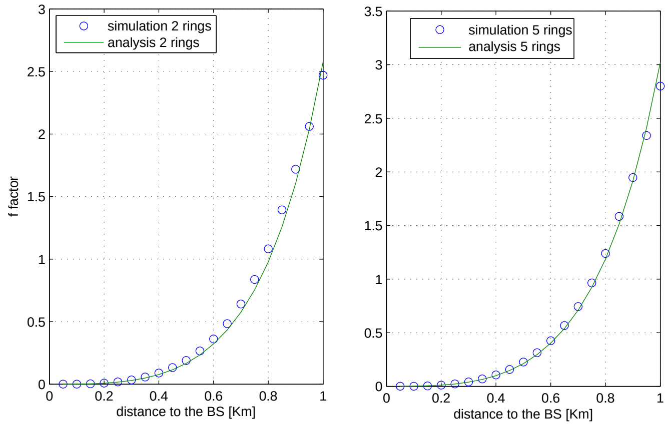

Fig. 5. OCIF vs. distance to the BS; comparison of the fluid model with simulations on a two ring (left) and a five ring (right) hexagonal network ($\eta = 3$).

network size. This model allows thus to develop analyses, adapted to each zone, taking into account each specific considered zone parameters.

We moreover note that our model can be used even for great distances between the base stations. We validate in Figure 6 the model considering two cell radii: a small one, $R = 500$ m and a large one, $R = 2$ Km. The latter curve allows us to conclude that our approach is accurate even for a very low base station density. It also shows that we can use the model for systems with frequency reuse different from one since in this case distances between co-channel BS are greater.

Figure 7 shows the dispersion of f_u at each distance for $\eta = 3$. For example, at $r_u = 1$ Km, f_u is between 2.8 and 3.3 for an average value of 3.0. This dispersion around the average value is due to the fact that in a hexagonal network, f_u is not isotropic.

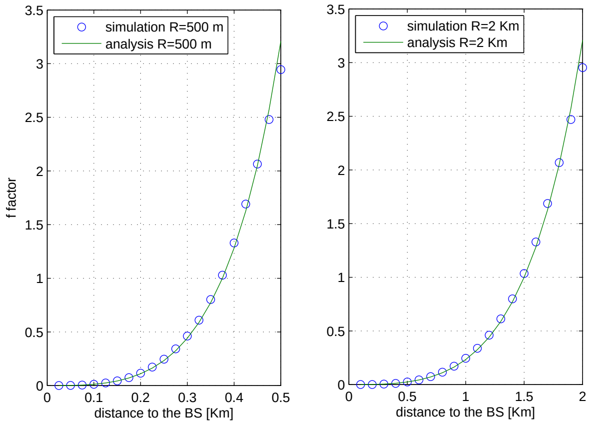

Fig. 6. OCIF vs. distance to the BS; comparison of the fluid model with simulations assuming cell radii $R = 500$ m and $R = 2$ Km ($\eta = 3$).

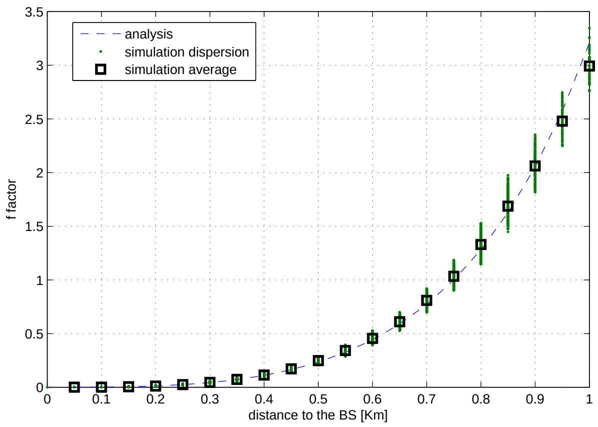

Fig. 7. OCIF vs. distance to the BS; comparison of the fluid model with simulations (average value and dispersion over 500 snapshots) on a hexagonal network with $\eta = 3$.

3.3 OCIF Formula for Hexagonal Networks

Two frameworks for the study of cellular networks are considered in this paper: the traditional hexagonal model and the fluid model. While the former is widely used, the latter is very simple and allows the derivation of an analytical formula for f_u. The last section has shown that both models lead to comparable results for the OCIF as a function of the distance to the BS. If we want to go further in the comparison of both models, in particular with the computation of outage probabilities, we need however to be more accurate.

Such calculations require indeed the use of the Q function (see section 4.3 and Eq.21 and 22), which is very sensitive to its arguments (mean and standard deviation). This point is rarely raised in literature: analysis and Monte Carlo simulations can lead to quite different outage probabilities even if analytical average and variance of the underlying Gaussian distribution are very close to simulated figures.

In this perspective, we provide an alternative formula for f_u that better matches the simulated figures in a hexagonal network. Note that this result is not needed if network designers use the new framework proposed in this paper.

We first note that f_u can be re-written in the following way:

$$f_u = \frac{\pi}{\sqrt{3}(\eta - 2)} x^{\eta} (2 - x)^{2-\eta}, \tag{15}$$

where $x = r_u/R_c$ and $\rho_{BS} = (2\sqrt{3}R_c^2)^{-1}$. As a consequence, f_u only depends on the relative distance to the serving BS, x, and on the path-loss exponent, η. For hexagonal networks, it is thus natural to find a correction of f_u that only depends on η. An accurate fitting of analytical and simulated curves shows that f_u should simply be multiplied by an affine function of η to match with Monte Carlo simulations in a hexagonal network. Eq.14 can then be re-written as follows:

$$f_{hexa,u} = (1 + A_{hexa}(\eta)) \frac{2\pi \rho_{BS} r_u^{\eta}}{\eta - 2} (2R_c - r_u)^{2-\eta}, \tag{16}$$

where $A_{hexa}(\eta) = 0.15\eta - 0.32$ is a corrective term obtained by least-square fitting. For example, $A_{hexa}(2.5) = 0.055$ (the correction is tiny) and $A_{hexa}(4) = 0.28$ (the correction is significative).

4 Outage Probabilities

In this section, we compute the global outage probability and the spatial outage probability with the Gaussian approximation. Closed-form formulas for the mean and standard deviation of f_u over a cell are provided.

Quality of service in cellular networks can be characterized by two main parameters: the blocking probability and the outage probability. The former is evaluated at the steady state of a dynamical system considering call arrivals and departures. It is related to a call admission control (CAC) that accepts or rejects new calls. The outage probability is evaluated in a semi-static system [15], where the number of MS is fixed and their locations are random. This approach is often used (see e.g. [16]) to model mobility in a simple way: MS jump from one location to another independently. For a given number of MS per cell, outage probability is thus the proportion of configurations, where the needed BS output power exceeds the maximum output power: $P_b > P_{max}$.

4.1 Global Outage Probability

For a given number of MS per cell, n, outage probability, $P_{out}^{(n)}$, is the proportion of configurations, for which the needed BS output power exceeds the maximum output power: $P_b > P_{max}$. If noise is neglected and if we assume a single service network ($\gamma_u^* = \gamma^*$ for all u), we deduce from Eq.8:

$$P_{out}^{(n)} = Pr\left[\sum_{u=0}^{n-1}(\alpha + f_u) > \frac{1-\varphi}{\beta}\right], \tag{17}$$

where $\varphi = P_{cch}/P_{max}$ and $\beta = \gamma^*/(1+\alpha\gamma^*)$.

4.2 Spatial Outage Probability

For a given number n of MS per cell, a spatial outage probability can also be defined. In this case, it is assumed that n MS have already been accepted by the system, i.e., the output power needed to serve them does not exceed the maximum allowed power. The spatial outage probability at location r_u is the probability that maximum power is exceeded if a new MS is accepted in r_u.

As for f_u, we make the approximation that the spatial outage, $P_{sout}^{(n)}(r_u)$, only depends on the distance to the BS and thus, can be written:

$$\begin{aligned} P_{sout}^{(n)}(r_u) &= Pr\left[(\alpha + f_u) + \sum_{v=0}^{n-1}(\alpha + f_v) > \frac{1-\varphi}{\beta} | \sum_{v=0}^{n-1}(\alpha + f_v) \leq \frac{1-\varphi}{\beta}\right] \\ &= \frac{Pr\left[\frac{1-\varphi}{\beta} - (\alpha + f_u) < \sum_{v=0}^{n-1}(\alpha + f_v) \leq \frac{1-\varphi}{\beta}\right]}{Pr\left[\sum_{v=0}^{n-1}(\alpha + f_v) \leq \frac{1-\varphi}{\beta}\right]} \end{aligned} \tag{18}$$

4.3 Gaussian Approximation

In order to compute these probabilities, we rely on the Central Limit theorem and use a Gaussian approximation. As a consequence, we need to compute the spatial mean and standard deviation of f_u. The area of a cell is $1/\rho_{BS} = \pi R_e^2$ with $R_e = R_c\sqrt{2\sqrt{3}/\pi}$. So, we integrate f_u on a disk of radius R_e. As MS are uniformly distributed over the equivalent disk, the probability density function (pdf) of r_u is: $p_{r_u}(t) = \frac{2t}{R_e^2}$. Let μ_f and σ_f be respectively the mean and standard deviation of f_u, when r_u is uniformly distributed over the disk of radius R_e.

$$\begin{aligned}
\mu_f &= \frac{2\pi\rho_{BS}}{\eta-2}\int_0^{R_e} t^{\eta}(2R_c-t)^{2-\eta}\frac{2t}{R_e^2}dt \\
&= \frac{2^{4-\eta}\pi\rho_{BS}R_c^2}{\eta-2}\left(\frac{R_e}{R_c}\right)^{\eta}\int_0^1 x^{\eta+1}\left(1-\frac{R_e x}{2R_c}\right)^{2-\eta}dx \\
&= \frac{2^{4-\eta}\pi\rho_{BS}R_c^2}{\eta^2-4}\left(\frac{R_e}{R_c}\right)^{\eta}{}_2F_1(\eta-2,\eta+2,\eta+3,R_e/2R_c),
\end{aligned} \tag{19}$$

where ${}_2F_1(a,b,c,z)$ is the hypergeometric function, whose integral form is given by:

$${}_2F_1(a,b,c,z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1 \frac{t^{b-1}(1-t)^{c-b-1}}{(1-tz)^a}dt,$$

and Γ is the gamma function.

Note that for $\eta = 3$, we have the simple closed formula:

$$\mu_f = -2\pi\rho_{BS}R_c^2\left(\frac{\ln(1-\nu/2)}{\nu^2}+\frac{16}{\nu}+4+\frac{4\nu}{3}+\frac{\nu^2}{2}\right),$$

where $\nu = R_e/R_c$. In the same way, the variance of $f(r)$ is given by:

$$\sigma_f^2 = E\left[f^2\right]-\mu_f^2 \tag{20}$$

$$E\left[f^2\right] = \frac{2^{4-2\eta}(2\pi\rho_{BS}R_c^2)^2}{(\eta+1)(\eta-2)^2}\left(\frac{R_e}{R_c}\right)^{2\eta}{}_2F_1(2\eta-4,2\eta+2,2\eta+3,\frac{R_e}{2R_c}).$$

As a conclusion of this section, the outage probability can be approximated by:

$$P_{out}^{(n)} = Q\left(\frac{\frac{1-\varphi}{\beta}-n\mu_f-n\alpha}{\sqrt{n}\sigma_f}\right), \tag{21}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} exp(-u^2/2)du$. And the spatial outage probability can be approximated by:

$$P_{sout}^{(n)}(r_u) = \frac{Q\left(\frac{\frac{1-\varphi}{\beta}-n\mu_f-(n+1)\alpha-f_u}{\sqrt{n}\sigma_f}\right)-Q\left(\frac{\frac{1-\varphi}{\beta}-n\mu_f-n\alpha}{\sqrt{n}\sigma_f}\right)}{1-Q\left(\frac{\frac{1-\varphi}{\beta}-n\mu_f-n\alpha}{\sqrt{n}\sigma_f}\right)}, \tag{22}$$

where f_u is given by Eq.14. This equation allows us to precisely compute the influence of an entering mobile station whatever its position in a cell and is thus the starting point for an efficient call admission control algorithm.

For cellular systems without internal interference, the definition of f_u is unchanged and Eq.21 and 22 are still valid provided that $\alpha = 0$.

Note that for an accurate fitting of the analytical formulas, which are presented in this section, to the Monte Carlo simulations performed in a hexagonal network, μ_f should be multiplied by $(1 + A_{hexa}(\eta))$, σ_f by $(1 + A_{hexa}(\eta))^2$ and Eq.14 replaced by Eq.16.

The question arises of the validity of the Gaussian approximation. The number of users per WCDMA (Wideband CDMA) cell is indeed usually not greater than some tens. Figure 8 compares the pdf of a gaussian variable with mean μ_f and standard deviation $\sigma_f/\sqrt{n}$ with the pdf of $\frac{1}{n}\sum_u f_u$ for different values for n. The latter pdf has been obtained by Monte Carlo simulations done on a single cell, assuming fluid model formula for f_u. We observe that gaussian approximation matches better and better when the number of mobiles increases. Even for very few mobiles in the cell ($n = 10$), the approximation is acceptable. So we can use it to calculate the outage probability.

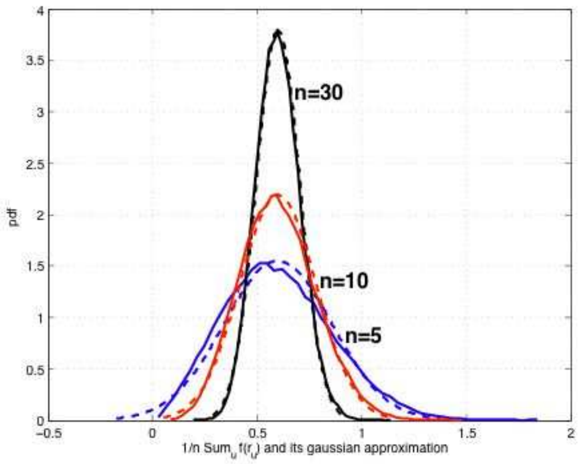

Fig. 8. Probability density function of $\frac{1}{n}\sum_u f_u$ (solid line) and its Gaussian approximation (dotted line).

4.4 Simulation Methodology

Monte Carlo simulations have been performed in order to validate the analytical approach. A fixed number n of MS are uniformly drawn on a given cell. All interferers are assumed to have the same transmitted power (homogeneous network). OCIF is computed according to Eq.5. Power transmitted by the cell is then compared to P_{max} for the calculation of the global outage probability.

For the spatial outage probability calculation, only snapshots without outage are considered. A new MS is added in the cell. The new transmitted power is again compared to P_{max} and the result is recorded with the distance of the new MS.

4.5 Results

Figures 9, 10 and 11 show some results we are able to obtain instantaneously using the simple formulas derived in this paper for voice service ($\gamma_u^* = -16$ dB). Analytical formulas are compared to Monte Carlo simulations in a hexagonal cellular network ($\alpha = 0.7$). Therefore, Eq.16 is used. Figure 9 shows the global outage probabilities as a function of the number of MS per cell for various values of the path-loss exponent η. It allows us to easily find the capacity of the network for any target outage. For example, a maximal outage probability of 10% leads to a capacity of about 16 users when $\eta = 3$. Figure 10 shows, as an example, the capacity with 2% outage as a function of η.

Figure 11 shows the spatial outage probability as a function of the distance to the BS for $\eta = 3$ and for various numbers of MS per cell. Given that there are already n, these curves give the probability that a new user, initiating a new call at a given distance, implies an outage. As an example, a new user in a cell with already 16 on-going calls, will cause outage with probability 10% at 550 m from the BS and with probability 20% at 750 m from the BS.

Traditional admission control schemes are based on the number of active MS in the cell. With the result of this paper, an operator would be able to admit or reject new connections also according to the location of the entering MS.

5 Application to Network Densification

In this section, we show the application of previous results to network densification. During the dimensioning process, the cell radius is determined by taking into account a maximum value of outage probability. This value characterizes the quality of service in terms of coverage the network operator wants to achieve. The number of BS to cover a given zone is directly derived from the cell radius.

Considering a maximum value of the outage probability, we first characterize cell breathing, i.e., the fact that cell coverage decreases when the cell load

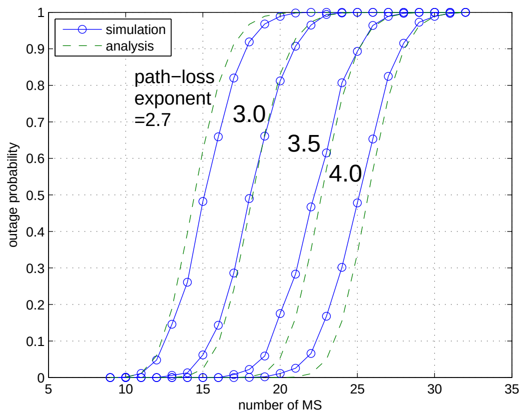

Fig. 9. Global outage probability as a function of the number of MS per cell and for path-loss exponents $\eta = 2.7$, 3.5 and 4, simulation and analysis.

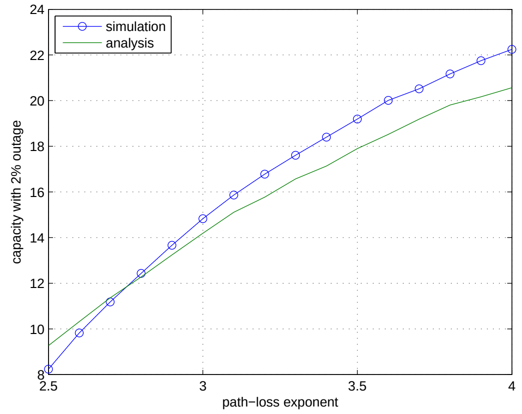

Fig. 10. Capacity with 2% outage as a function of the path-loss exponent η, simulations (solid lines) and analysis (dotted lines) are compared.

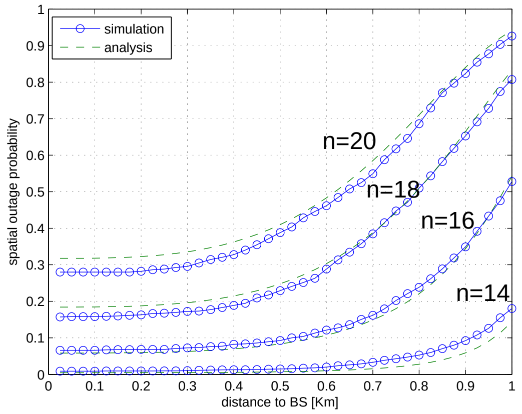

Fig. 11. Spatial outage probability as a function of the distance to the BS for various numbers of users per cell and for $\eta = 3$.

increases. We then analyze BS densification as an answer to cell breathing.

5.1 Cell Breathing Characterization

Let consider a maximum value of outage probability $P^*_{out} = P^{(n)}_{out}$. From Eq.21, we can write:

$$Q^{-1}(P^*_{out}) = \frac{\frac{1-\varphi}{\beta} - n\mu_f - n\alpha}{\sqrt{n}\sigma_f}. \tag{23}$$

Denoting $a = \frac{1-\varphi}{\beta}$, that equation can be expressed as:

$$(\alpha + \mu_f)^2 n^2 - \left(2a(\alpha + \mu_f) + \sigma_f^2 Q^{-1}(P^*_{out})^2\right) n + a^2 = 0. \tag{24}$$

As ρ_{MS} is the mobile density, we can write $n = \rho_{MS} A_{cov}$, where n is the maximum number of mobiles served by a BS for maximum outage probability P^*_{out}, and A_{cov} is the area covered by the BS. Let $A_{cell} = 2\sqrt{3}R_c^2 = 1/\rho_{BS}$ be the cell area. When mobile density increases, A_{cov} decreases, so that $A_{cov} \leq A_{cell}$.

We now obtain the following equation:

$$(\alpha + \mu_f)^2 A_{cov}^2 \rho_{MS}^2 - \left(2a(\alpha + \mu_f) + \sigma_f^2 Q^{-1}(P^*_{out})^2\right) A_{cov}\rho_{MS} + a^2 = 0 \tag{25}$$

This equation has two solutions. The maximum mobile density can be expressed as:

$$\rho_{MS} = \frac{1}{2(\alpha + \mu_f)^2 A_{cov}} \Big\{ 2a(\alpha + \mu_f) + \sigma_f^2 Q^{-1}(P^*_{out})^2 + \sqrt{\sigma_f^2 Q^{-1}(P^*_{out})^2 (\sigma_f^2 Q^{-1}(P^*_{out})^2 + 4a(\alpha + \mu_f))} \Big\}. \tag{26}$$

In this equation, mean and standard deviation of f_u, μ_f and σ_f, are computed over the covered area $\mathcal{A}_{cov}$ with surface A_{cov}:

$$\mu_f = \frac{1}{A_{cov}} \int_{\mathcal{A}_{cov}} f_u ds \tag{27}$$

$$\sigma_f^2 = \frac{1}{A_{cov}} \int_{\mathcal{A}_{cov}} f_u^2 ds - \mu_f^2. \tag{28}$$

Eq.26 shows the link between the mobile density and the covered area and is now used to characterize cell breathing.

Numerical values in Figure 12 shows the results we obtain thanks to Eq.26 assuming voice service ($\gamma_u^* = -16$ dB), $\varphi = 0.2$, $\alpha = 0.7$, $\eta = 3$ in a CDMA network.

The solid curve shows the mobile density as a function of the coverage area of base stations. On this curve, the BS density is supposed to be constant, $R_c = 1$ Km and thus $A_{cell} = 2\sqrt{3} \approx 3.46$ Km^2. The coverage area A_{cov} however shrinks when the traffic (characterized here by the density of mobiles ρ_{MS}) increases. For example, going from point 1 with $\rho_{MS} = 10$ mobiles/Km^2 to point 2 with $\rho_{MS} = 15$ mobiles/Km^2 reduces the covered area from 3.46 Km^2 to approximately 2.6 Km^2. As a consequence, the cell is not completely covered, and due to cell breathing, coverage holes appear.

5.2 Base Station Densification

A way of solving the issue of cell breathing is to densify the network. The dotted line in Figure 12 plots the mobile density as a function of the covered area assuming full coverage of the cell. Along this curve, $A_{cov} = A_{cell}$ and when A_{cov} is decreasing, the BS density is increasing. We are thus able to find, for a given mobile density, the BS density that will ensure continuous coverage in the network. For example, for 20 mobiles/Km^2, the cell area should be approximately 1.5 Km^2 in order to avoid coverage holes.

The sequence 1-2-3 shows an example of scenario, where BS densification is needed. In point 1, the network has been dimensioned for 10 mobiles/Km^2. If the cellular operator is successful and so more subscribers are accessing the network, mobile density increases along the solid line of Figure 12. At point 2, coverage holes appear and the operator decides to densify the network. While adding new BS, he has to jump to point 3 in order to ensure continuous coverage. At point 2, he needs approximately 0.38 BS per Km^2 ($A_{cov} = 2.6$ Km^2), while at point 3, he needs about 0.48 BS per Km^2 ($A_{cov} = 2.1$ Km^2), which corresponds to a 26% increase.

6 Conclusion

In this paper, we have proposed and validated by Monte Carlo simulations a fluid model for the estimation of outage and spatial outage probabilities in cellular networks. This approach considers BS as a continuum of transmit-

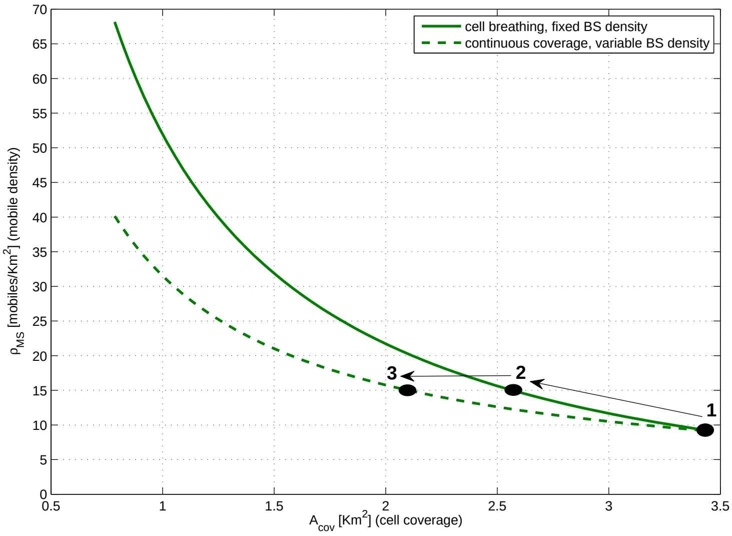

Fig. 12. Cell breathing (fixed BS density) and BS densification (variable BS density) in a cellular network.

ters and provides a simple formula for the other-cell power ratio (OCIF) as a function of the distance to the BS, the path-loss exponent, the distance between BS and the network size. Simulations show that the obtained closed-form formula is a very good approximation, even for the traditional hexagonal network. The simplicity of the result allows a spatial integration of the OCIF leading to closed-form formula for the global outage probability and for the spatial outage probability. At last, this approach allows us to quantify cell breathing and network densification.

References

[1] A. M. Viterbi and A. J. Viterbi, Erlang Capacity of a Power Controlled CDMA System, IEEE Journal on Selected Areas in Communications, Vol. 11, No. 6, Aug. 1993.

[2] A. J. Viterbi, A. M. Viterbi, and E. Zehavi, Other-Cell Interference in Cellular Power-Controlled CDMA, IEEE Trans. on Communications, Vol. 42, No. 2/3/4, Feb./Mar./Apr. 1994.

[3] A. J. Viterbi, CDMA - Principles of Spread Spectrum Communication, Addison-Wesley, 1995.

[4] X. Lagrange, Principes et évolutions de l'UMTS, Hermes, 2005.

[5] J. S. Evans and D. Everitt, Effective Bandwidth-Based Admission Control for Multiservice CDMA Cellular Networks, IEEE Trans. on Vehicular Technology, Vol. 48, No. 1, Jan. 1999.

[6] T. Liu and D. Everitt, Analytical Approximation of Other-Cell Interference in Uplink of CDMA Cellular Systems, Proc. of IEEE VTC Spring, May 2006.

[7] S. E. Elayoubi and T. Chahed, Admission Control in the Downlink of WCDMA/UMTS, Lect. Notes Comput. Sci., Springer, 2005.

[8] K. S. Gilhousen, I. M. Jacobs, R. Padovani, A. J. Viterbi, L. A. Weaver, and C. E. Wheatley, On the Capacity of Cellular CDMA System, IEEE Trans. on Vehicular Technology, Vol. 40, No. 2, May 1991.

[9] C. C. Chan and S. V. Hanly, Calculating the Outage Probability in CDMA Network with Spatial Poisson Traffic, IEEE Trans. on Vehicular Technology, Vol. 50, No. 1, Jan. 2001.

[10] J.-M. Kelif, Admission Control on Fluid CDMA Networks, Proc. of WiOpt, Apr. 2006.

[11] J-M. Kelif and E. Altman, Downlink Fluid Model of CDMA Networks, Proc. of IEEE VTC Spring, May 2005.

[12] J-M. Kelif, M. Coupechoux, and Ph. Godlewski, Spatial Outage Probability for Cellular Networks, Proc. of IEEE Globecom, Nov. 2007.

[13] J-M. Kelif, M. Coupechoux, and Ph. Godlewski, Fluid Model of the Outage Probability in Sectored Wireless Networks, Proc. of IEEE WCNC, Mar. 2008.

[14] J.-M. Kelif, Modèle fluide de réseau sans fils, Thésis, ENST Paris, Feb 2008.

[15] F. Baccelli, B. Blaszczyszyn, and M. K. Karray, Blocking Rates in Large CDMA Networks via a Spatial Erlang Formula, Proc. of IEEE Infocom, Mar. 2005.

[16] T. Bonald and A. Proutiere, Conservative Estimates of Blocking and Outage Probabilities in CDMA Networks, Proc. of Performance 2005.

[17] S. Toumpis and L. Tassiulas, Packetostatics Deployment of Massively Dense Sensor Networks as an Electrostatics Problem, Proc. of IEEE Infocom, Mar. 2005.

[18] P. Jacquet, Geometry of Information Propagation in Massively Dense Ad hoc Networks, Proc. of ACM MobiHoc, May 2004.

[19] T. Bonald and A. Proutiere, Wireless Downlink Data Channels: User Performance and Cell Dimensioning, Proc. of ACM Mobicom 2003.