

# Load Balancing in Heterogeneous Networks Based on Distributed Learning in Near-Potential Games

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**Abstract**—We present a novel approach for distributed load balancing in heterogeneous networks that use cell range expansion (CRE) for user association and almost blank subframe (ABS) for interference management. First, we formulate the problem as a minimisation of an  $\alpha$ -fairness objective function with load and outage constraints. Depending on  $\alpha$ , different objectives in terms of network performance or fairness can be achieved. Next, we model the interactions among the base stations for load balancing as a near-potential game, in which the potential function is the  $\alpha$ -fairness function. The optimal pure Nash equilibrium (PNE) of the game is found by using distributed learning algorithms. We propose log-linear and binary log-linear learning algorithms for complete and partial information settings, respectively. We give a detailed proof of convergence of learning algorithms for a near-potential game. We provide sufficient conditions under which the learning algorithms converge to the optimal PNE. By running extensive simulations, we show that the proposed algorithms converge within few hundreds of iterations. The convergence speed in the case of partial information setting is comparable to that of the complete information setting. Finally, we show that outage can be controlled and a better load balancing can be achieved by introducing ABS.

## I. INTRODUCTION

Due to the ever increasing demand for improved quality of service in terms of higher data rates and improved coverage, the conventional cellular networks are becoming heterogeneous. Heterogeneous networks consist of macro base stations (BSs) and small BSs that transmit with high and low power, respectively. Conventional user association rule is such that the users select a BS that provides the highest received power. This may however result in an imbalance between BSs loads because the macro BSs transmit at higher power and thus associates with more users. This creates overload situation at the macro BSs and at the same time under-utilised resources at the small BSs. Therefore, a natural problem that arises is how to associate users to BSs such that the network resources are utilised efficiently and the load is shared among the BSs.

Load balancing has been extensively studied in the literature using various approaches. An overview can be found in [2], [3]. These can be broadly classified as centralised, e.g. in [4]–[7], and decentralised optimisation approaches, see e.g. [8]–[12]. However, centralised solutions are computationally extensive, require huge information exchange overhead, and are

thus not scalable. To overcome these limitations, decentralised approaches have been proposed. Load balancing is modeled as a convex optimisation problem in [8] and a distributed algorithm, which is a fixed point iteration, is proposed to solve it. It is a user-centric approach, in which users decide to which BS they associate based on load information broadcast by BSs. User-centric game theoretical and learning approaches are also proposed, e.g. using congestion games [9], [11], evolutionary games [10] or distributed Q-learning [12].

Some network-centric approaches include power control and cell range expansion (CRE). In power control more users can be offloaded to small BSs by increasing their transmit power because user association is based on maximum received power of user [13]. Since, small BSs have tight power constraint, power control may not be feasible for efficient load balancing. In this paper, we focus on an alternative network-centric approach, in which BSs take decisions and users follow a predefined association rule called CRE. According to the CRE technique, users associate with a BS that provides the maximum biased received power. A CRE bias is broadcast by every BS and is typically higher for small BSs than for macro BSs. This results in an increase of the small cell coverage and thereby of the number of users associated to them. CRE technique has the drawback of increasing outage probability at the cell edge [14], it is therefore often deployed in conjunction with almost blank subframes (ABS) at the macro BS [15]. During these subframes that represent a fixed ratio of the radio frame, macro BSs drastically lower their transmit power, so that small BSs cell edge users can experience less interference, when scheduled during these periods.

The challenge we intend to tackle here is to jointly determine the optimal CRE bias values and ABS ratios for a required optimal performance of the network. Several papers try to achieve a similar goal. In a first set of papers, performance evaluation and optimisation are performed using simulations or experiments [16]–[18]. They give interesting insights but provide optimal values only for some specific scenarios. Centralised approaches provide upper bounds on performance but fail to address the scalability issue, see e.g. [19]–[23]. In [20] for example, authors formulate an optimisation problem aiming at maximising the Jain’s fairness index between station loads.

Another set of papers is focused on distributed algorithms [12], [19], [24]–[27]. Early papers [19], [24], [25] propose heuristics without any goal of achieving some kind of optimality. In [27], authors formulate an integer programming problem, relaxed into a convex problem, and they propose a distributed algorithm based on Lagrangian dual decom-

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position. ABS and related trade offs are however ignored. In [26], two independent algorithms are presented for optimal CRE bias and ABS ratio, respectively. Optimal parameters are obtained for a given number of active users and the outage constraint is ignored. In a recent paper [28], Liu et al. model the system as a potential game and use best response algorithm to reach a Nash Equilibrium (NE) that is also a local optimiser of a proportional fairness objective function. In this paper, as in others (e.g. [23], [26], [27]), a static *full buffer* traffic model is assumed. This implies that proposed distributed algorithms should converge faster than the change of traffic conditions. This is an assumption that does not seem realistic for practical implementations. On the contrary, we assume a dynamic traffic model and we update association parameters after the traffic has reached a stationary state. This is also the approach of [8], [18], which respectively focus on user-centric load balancing (without CRE or ABS) and a centralised approach. Very few analytical works (with the exception of [12]) consider outage probability as a possible constraint when using CRE. Nowhere in the above literature, the effect of shadowing is studied in conjunction with CRE and ABS.

In this paper, we propose a general framework for determining jointly the optimal CRE biases and the optimal ABS ratios for different performance requirements of the network. We address this problem by considering an  $\alpha$ -fairness objective function that captures various aspects of the network performance and fairness for different  $\alpha$  values. A similar function is used in [8] but in a different context with the main difference that our problem is not convex. We model our system using the notion of near-potential game [29], a framework needed to take into account shadowing in the radio propagation model. We solve the non-convex optimisation problem using distributed learning algorithms, which converge in probability to the best NE of the game even in absence of complete information (in contrast with [28]).

### A. Contributions

*Novel approach:* We present a novel approach for load balancing in heterogeneous networks that uses CRE for user association and ABS for interference coordination. Our approach is to distributively minimise an  $\alpha$ -fairness objective function using distributed learning algorithms in near-potential games with load and outage constraints. We assume a dynamic traffic model and a time scale separation between traffic dynamics and user association parameters update.

*Non-convex constrained optimisation:* First, we model the load balancing problem as a non-convex constrained optimisation with overload and outage probability constraints. The  $\alpha$ -fairness objective function captures various network performances and fairnesses for different  $\alpha$ . For  $\alpha = 0$  the objective function captures min-sum-load policy of the network. For  $\alpha \rightarrow \infty$ , we prove that it results in the min-max load policy (Theorem 1). We extend the classical result derived in [30] by considering a non-convex  $\alpha$ -fairness function. Optimal CRE biases and ABS ratios are the solution set.

*Near-potential game framework:* Then, we model the system as a near-potential game (c.f. [29]) using a simple utility

structure, which only requires the knowledge of the neighborhood of every BS (similar idea in [28]). In presence of shadowing the neighborhood of every BS may include the whole network. As a consequence, the potential game proposed in [28] can only be solved in a centralised way. Relying on near-potential game we solve the problem distributively.

*Learning algorithms:* By adapting log-linear learning algorithms (LLLA) to outage and load constraints, we achieve the global minimum of the objective function. We consider two different settings: complete and partial information. In the former setting, we adapt classical LLLA, whereas in the latter setting, we adapt binary LLLA (BLLLA). We prove the convergence of LLLA and BLLLA in near-potential games to an  $\epsilon$ -NE, whose potential is closed to the global minimum of the objective function (Theorem 3). This result is also proved in [29]. We however extend to a more general framework by using a different proof technique. Our technique does not need stationary revision process. Instead the revision process can be state and history dependent. Next, we provide sufficient conditions on the parameter that controls the size of the neighborhood, for convergence to an  $\epsilon$ -NE with potential close to the global optimum of the objective function (Theorem 4). We also provide stronger conditions for the convergence to the optimal PNE (Corollary 3). To the best of our knowledge, these conditions are not present in the literature. Finally, we propose a practical step by step construction of the neighborhood with guaranteed convergence based on mobile users measurements (Section IV-E).

*Numerical results:* By running extensive simulations, we show that the proposed algorithms converge within few hundreds of iterations to the global minimum. The convergence speed of the BLLLA is comparable to that of the LLLA, meaning that partial information is sufficient in practical implementations. We show that the load balancing problem can be solved distributively by restricting the number of base stations neighbours. Numerical results show that for load balancing LLLA and BLLLA outperform the algorithm in [31]. We show that outage can be controlled without ABS but at the price of undermining the interest of using CRE technique. The introduction of ABS allows for low outage together with better load balancing.

This paper is organised as follows. In Section II, the system model is described and the problem is formulated. In Section III, a near-potential game framework solution is presented. The various distributed algorithms that are considered in this paper are described in Section IV. In Section V, our approach is validated using extensive simulations. Finally, the conclusions are given in Section VI.

## II. SYSTEM MODEL

### A. Network Model

We consider the downlink<sup>1</sup> of a cellular network (typically a LTE-Advanced network) consisting of a set  $\mathcal{B}_e$  of macro BSs (typically eNodes-B) and a set  $\mathcal{B}_s$  of small BSs in a two

<sup>1</sup>Downlink is usually considered as the dominant link in terms of traffic. However, optimal user association on the downlink may not be optimal for the uplink [32].

dimensional region  $\mathcal{L}$ . The set of all stations is denoted  $\mathcal{S} \triangleq \mathcal{B}_e \cup \mathcal{B}_s$ . The transmit powers of macro and small BSs are denoted as  $P_{\text{macro}}$  and  $P_{\text{small}}$ , respectively. There are special subframes called ABS, during which a macro BS transmits with reduced power  $P_{\text{ABS}}$ . The proportion of ABS subframes is denoted  $\theta_i \in [0; 1]$  for BS  $i$ . Let  $\bar{\theta} = [\theta_1, \theta_2, \dots, \theta_{|\mathcal{S}|}]$  be the ABS ratio vector. We assume that small BSs do not implement ABS technique, i.e.,  $\theta_i = 0$  for  $i \in \mathcal{B}_s$ . Every small BS  $i$  maintains a parameter  $c_i \in [1; c_{\text{max}}]$  called CRE bias. The CRE bias vector is denoted  $\bar{c} = [c_1, c_2, \dots, c_{|\mathcal{S}|}]$ . The CRE biases for macro BSs are fixed to unity, i.e.,  $c_k = 1, \forall k \in \mathcal{B}_e$ . This leads to no bias in the received power from a macro BS.

1) *Channel Model*: The received power at location  $x$  from BS  $i$  is  $P_i g_i(x)$ , where  $P_i$  is the transmit power and  $g_i(x)$  is the channel gain, which captures the effect of path-loss and shadowing. The effect of small-scale fading is not considered because the time for user association procedure is assumed to be much larger than the channel coherence time [8]. We consider a scenario where the locations of the BSs and of the users during their download are fixed. Therefore, the shadow fading component is a constant multiplicative factor. Formally, the channel gain model considered is [33]:

$$g_i(x) = \min \left\{ 1, K |x - x_i|^{-\eta} e^{\beta y_i(x)} \right\}, \quad (1)$$

where  $K = \left( \frac{\lambda_w}{4\pi d_0} \right)^2$ ,  $\lambda_w$  is the wavelength,  $d_0$  is the reference distance,  $x_i$  is the location of the BS  $i$ ,  $\eta \geq 2$  is the path-loss exponent, and  $e^{\beta y_i(x)}$  is the shadowing component where  $\beta = \frac{\log 10}{10}$  and  $y_i(x)$  is a realisation of Gaussian random process of zero mean and covariance function  $C_{y_i}(\Delta x)$  [34]:

$$C_{y_i}(\Delta x) = \sigma_{sh}^2 e^{-\frac{\Delta x}{D_c}}, \quad (2)$$

where  $\sigma_{sh}^2$  is the variance,  $\Delta x$  is the displacement, and  $D_c$  is the decorrelation distance [33]. A constant cross correlation between the  $y_i(x)$  and  $y_j(x)$  is considered as in [35].

Let  $M$  be the number subframes in a given radio frame. For every allowed value of  $\theta$ , we assume that there is a fixed ABS pattern  $\Upsilon(\theta)$ , i.e., a set of subframes during which a BS transmits at lower power. Notice that  $\Upsilon(0) = \emptyset$ . Then the SINR  $\gamma_i^f(x, \bar{\theta})$  of a user at location  $x$  in a subframe  $f$  is given as:

$$\gamma_i^f(x, \bar{\theta}) = \frac{P_i^f(\theta_i) g_i(x)}{\sum_{j \in \mathcal{S}} P_j^f(\theta_j) g_j(x) + N_0}, \quad (3)$$

where

$$P_i^f(\theta_i) = \begin{cases} P_{\text{ABS}} & \text{if } f \in \Upsilon(\theta_i), i \in \mathcal{B}_e, \\ P_{\text{macro}} & \text{if } f \notin \Upsilon(\theta_i), i \in \mathcal{B}_e, \\ P_{\text{small}} & \text{otherwise} \end{cases} \quad (4)$$

and  $N_0 = -174 + 10 \log W$  is thermal noise power in dBm and  $W$  is system bandwidth in Hz.

2) *CRE User Association Rule*: A user association rule based on CRE and maximum transmit power is commonly used in the heterogeneous networks [2], [14]–[16], [36]–[42]. According to this rule, a user located at  $x$  is associated to the BS  $i$  that provides the highest biased received power. The set of locations  $\mathcal{D}_i(\bar{c})$  associated to BS  $i$  is defined as:

$$\mathcal{D}_i(\bar{c}) = \{x | \forall j \in \mathcal{S}, P_i g_i(x) c_i \geq P_j g_j(x) c_j\}, \quad (5)$$

where  $P_i = P_{\text{macro}}$  if  $i \in \mathcal{B}_e$  and  $P_{\text{small}}$  otherwise.

3) *Physical Data Rate*: The physical data rate received by a user at  $x$  in a subframe  $f$  when it is served by BS  $i$  is denoted  $\tilde{\nu}_i^f(x, \bar{\theta})$ . The user average data rate over a radio frame is  $\nu_i(x, \bar{\theta}) = \frac{1}{M} \sum_{f=1}^M \tilde{\nu}_i^f(x, \bar{\theta})$ . This should be understood as the throughput achievable by the user alone in its cell. The function  $\tilde{\nu}_i^f(x, \bar{\theta})$  is a non-negative and non-decreasing function of the SINR  $\gamma_i^f(x, \bar{\theta})$ . For SINR below minimum threshold  $\gamma_{\text{min}}$  the user is not served and  $\tilde{\nu}_i^f(x, \bar{\theta}) = 0$ .

4) *Traffic Model*: Users are assumed to arrive in the system according to a spatial random process, download a file of random size and leave the system when the download is over. This is referred to as elastic traffic. All users are scheduled in all subframes<sup>2</sup> [26]. At location  $x$ , the arrival rate is denoted  $\lambda(x)$  [arrivals/s/m<sup>2</sup>] and the average file size is  $1/\mu(x)$  [bits]. Following [8], we model every BS  $i$  as a M/G/1/PS queue of load:

$$\rho_i(\bar{c}, \bar{\theta}) = \int_{\mathcal{D}_i(\bar{c})} \frac{\lambda(x)}{\mu(x) \nu_i(x, \bar{\theta})} 1_{\{\max_f \gamma_i^f(x, \bar{\theta}) \geq \gamma_{\text{min}}\}} dx. \quad (6)$$

BS  $i$  is stable if and only if  $0 \leq \rho_i < 1$ . In this work, only stable network configurations are considered.

*Assumption 1*: [Time-scale separation] The process of updating the CRE bias and ABS ratio is supposed to be long with respect to the traffic variations. The M/G/1/PS queues describing the BSs traffic are thus supposed to have reached their stationary regime before any new change of these parameters.

*Outage Probability* is defined as the fraction of users that are not served. Recall that a user is not served if its SINR is below minimum threshold  $\gamma_{\text{min}}$ . Formally, the outage probability  $O_i$  observed by BS  $i$  is given by:

$$O_i(\bar{c}, \bar{\theta}) = \frac{\int_{\mathcal{D}_i(\bar{c})} \lambda(x) 1_{\{\max_f \gamma_i^f(x, \bar{\theta}) < \gamma_{\text{min}}\}} dx}{\int_{\mathcal{D}_i(\bar{c})} \lambda(x) dx}. \quad (7)$$

In this definition, as soon as there is at least one subframe during which the SINR is above the threshold, the user is supposed to be served.

## B. Problem Formulation and Objective Function

Following [8], we intend to minimise an  $\alpha$ -fairness function  $\phi_\alpha(\bar{c}, \bar{\theta})$  over a feasible set  $\mathcal{F}$ , which are defined as:

$$\phi_\alpha(\bar{c}, \bar{\theta}) = \begin{cases} \sum_{i \in \mathcal{S}} \frac{(1 - \rho_i(\bar{c}, \bar{\theta}))^{1-\alpha}}{\alpha - 1}, & \alpha \geq 0, \alpha \neq 1, \\ -\sum_{i \in \mathcal{S}} \log(1 - \rho_i(\bar{c}, \bar{\theta})), & \alpha = 1, \end{cases} \quad (8)$$

$$\mathcal{F} = \{ \{\bar{c}, \bar{\theta}\} | \forall i \in \mathcal{S}, \rho_i(\bar{c}, \bar{\theta}) < 1, O_i(\bar{c}, \bar{\theta}) < \bar{O}_i \}, \quad (9)$$

where  $\bar{O}_i$  is the maximum outage probability for BS  $i$ . The function  $\phi_\alpha(\bar{c}, \bar{\theta})$  is in general non-convex and even if it is convex the set  $\mathcal{F}$  is non-convex because  $\bar{c}$  takes discrete values. The function  $\phi_\alpha(\bar{c}, \bar{\theta})$  captures various aspects of fairness and performance for the network depending on the choice of  $\alpha$ .

<sup>2</sup>Another scheduling policy where only those users are served that are in the extended region can also be included in our model [26]. However, it adds more complexity without affecting the conclusions of this work.

( $\alpha = 0$ ) **Min-sum-load policy:** Minimising  $\phi_0(\bar{c}, \bar{\theta})$  minimizes the sum of BSs loads in general. In the particular case, where  $\bar{\theta} = 0$ , it results in a rate-optimal policy (see Appendix A). ( $\alpha = 1$ ) **Proportional fair policy:** Minimising  $\phi_1(\bar{c}, \bar{\theta})$  is equivalent to achieving proportional fairness between BSs [30]. ( $\alpha = 2$ ) **Delay-optimal policy:** It can be shown that minimising  $\phi_2(\bar{c}, \bar{\theta})$  is equivalent to minimising the average throughput of the network. The average throughput of a stable M/G/1/PS queue is the product of arrival rate and average delay. In our system model, the arrival rate is independent of CRE bias and ABS ratio. Therefore, minimising  $\phi_2(\bar{c}, \bar{\theta})$  is equivalent to minimising the average delay of the network. For more detailed discussion refer to [8]. ( $\alpha \rightarrow \infty$ ) **Minmax policy:** As  $\alpha \rightarrow \infty$  the minimiser of  $\phi_\alpha(\bar{c}, \bar{\theta})$  tends to the min-max load vector. It is a standard result with convex objective function [8], [30], [43]. We now prove this result for our non-convex objective function in Theorem 1.

*Definition 1:* [Min-max load vector [43]] Let all the vectors in  $\mathcal{F}$  be sorted in increasing order. A vector  $\rho \in \mathcal{F}$  is min-max if  $\rho$  is lexicographically not greater than any vector in  $\mathcal{F}$ . The vector  $\rho$  is *lexicographically lower* than  $y$ , denoted  $\rho \prec y$ , if the first non-zero component of  $\rho - y$  is negative. We say that  $\rho$  is not greater than  $y$ , denoted by  $\rho \preceq y$ , if  $\rho \prec y$  or  $\rho = y$ .

Let  $r_i(\bar{c}) = 1 - \rho_i(\bar{c}, \bar{\theta}), \forall i \in \mathcal{S}$ . Let  $\mathcal{X} = \{r \in \mathcal{R}^{|\mathcal{S}|} | \exists \bar{c} : \rho(\bar{c}) \in \mathcal{F}, r(\bar{c}) = r\}$ . Load vector  $\rho^*$  is a min-max if and only if  $r^*$  is max-min vector.

**Theorem 1:** Let  $r_\alpha \in \arg \max_{r \in \mathcal{X}} \sum_{i \in \mathcal{S}} \frac{r_i^{1-\alpha}}{1-\alpha}$ . Then, any accumulation vector of the trajectory  $\{r_\alpha\}_{\alpha > 1}$  is max-min in  $\mathcal{X}$ .

*Proof:* See Appendix B. ■

In Fig. 1, we show an example of  $\mathcal{F}$  set obtained with 2 BSs having different transmit powers located on a two-dimensional region. It is clear from the figure that even if the CRE set were continuous,  $\mathcal{F}$  would not be convex. We also show the optimal loads obtained for different  $\alpha$  values. All the optimal load points are located on the Pareto frontier. The point for  $\alpha \geq 200$  in Fig. 1 is the min-max load point because a point of equal coordinates on the Pareto frontier is the min-max point.

### III. NEAR-POTENTIAL GAME FRAMEWORK

In this section, we present an approach using near-potential game framework for distributed optimisation of the objective function. We do not intend to describe and analyse selfish nature of BSs that aim to minimise their costs. Rather, our goal is to achieve the global objective of load balancing by prescribing a cost function to the BSs. For this context, potential games provide a good framework because players of such a game distributively optimise a potential function.

We model the problem as a *user association game*, where the BSs are players and allowed CRE bias and ABS ratio values are their strategies. BSs play the user association game with the objective of minimising their costs. An  $\epsilon$ -NE of the game is reached when no player can benefit more than  $\epsilon$  by changing its strategy unilaterally.

*Definition 2:* [ $\epsilon$ -Nash equilibrium] Let  $\mathcal{G} = \{\mathcal{S}, \{X_i\}_{i \in \mathcal{S}}, \{U_i\}_{i \in \mathcal{S}}\}$  be a game, where  $\mathcal{S}$  is set of players,  $\{X_i\}_{i \in \mathcal{S}}$  are strategy sets, and  $\{U_i\}_{i \in \mathcal{S}}$  are cost functions. Let  $a_i$  be a strategy profile of player  $i$  and  $a_{-i}$  be

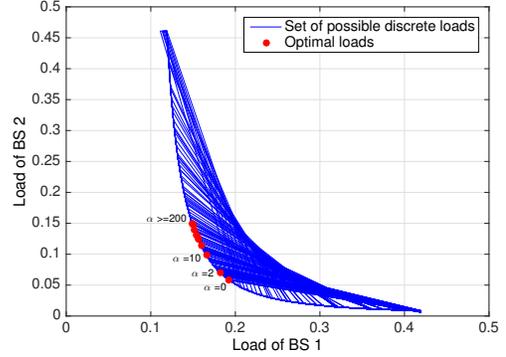


Fig. 1: Feasible set  $\mathcal{F}$  for 2 BSs.

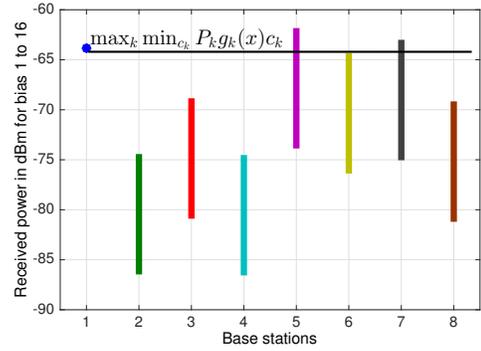


Fig. 2: Illustration of the neighbour set.

a strategy profile of all players except for player  $i$ . A strategy profile  $(a_i^*, a_{-i}^*)$  is an  $\epsilon$ -NE if

$$U_i(a_i^*, a_{-i}^*) - U_i(a_i, a_{-i}^*) \leq \epsilon, \quad \forall a_i \in X_i, \forall i \in \mathcal{S}. \quad (10)$$

If  $\epsilon = 0$  then it is a pure Nash equilibrium (PNE).

Based on the notion developed in [29] we now define  $\epsilon$ -potential game.

*Definition 3:* [ $\epsilon$ -potential game] A game  $\mathcal{G} = \{\mathcal{S}, \{X_i\}_{i \in \mathcal{S}}, \{U_i\}_{i \in \mathcal{S}}\}$  is an  $\epsilon$ -potential game if there is a potential function  $h : X \rightarrow \mathcal{R}$  such that  $\forall i \in \mathcal{S}, \forall a_i, a'_i \in X_i$  and  $\forall a_{-i} \in X_{-i}$ ,

$$|U_i(a_i, a_{-i}) - U_i(a'_i, a_{-i}) + h(a'_i, a_{-i}) - h(a_i, a_{-i})| \leq \epsilon. \quad (11)$$

For  $\epsilon = 0$ , it is an exact potential game [44]. The  $\epsilon$  captures the *maximum pairwise difference* between an  $\epsilon$ -potential game and an exact potential game with the same potential function as in [29, Definition 2.2].

An exact potential game has at least one PNE and local optimisers of the potential function are PNEs [44]. In the following lemma, we provide the relationship between the PNEs of a potential game and a near-potential game with the same potential.

**Lemma 1:** Let  $\mathcal{G} = \{\mathcal{S}, \{X_i\}_{i \in \mathcal{S}}, \{U_i\}_{i \in \mathcal{S}}\}$  and  $\mathcal{G}' = \{\mathcal{S}, \{X_i\}_{i \in \mathcal{S}}, \{U'_i\}_{i \in \mathcal{S}}\}$  be an exact potential game and an  $\epsilon$ -potential game respectively, sharing a common potential function. If  $a^*$  is a PNE for  $\mathcal{G}$  then it is an  $\epsilon$ -NE for  $\mathcal{G}'$ .

*Proof:* See Appendix C. ■

In our problem, we seek that the objective function (8) is turned into a potential function of the user association game. The issue is in designing the cost functions of the BSs to obtain

an  $\epsilon$ -potential game, where  $\epsilon$  represents a trade off between the quality of the solution and the distributed nature of the algorithm. We consider a simple cost structure for BS  $i$ , which takes into account only the effects of its neighbours. A similar approach is used to obtain an exact potential game in [45], [46]. The cost functions of the individual BSs are defined as:

$$U_i^\varpi(a_i, a_{-i}) = \sum_{j \in N_i^\varpi} \frac{(1 - \rho_j(a_i, a_{-i}))^{1-\alpha}}{\alpha - 1}, \quad (12)$$

where  $N_i^\varpi$  is the neighborhood of BS  $i$ ,  $\varpi$  is a parameter to control its size, and  $\rho_j(a_i, a_{-i})$  is the load of BS  $j$  given in (6). With this cost function, we now formally define the user association game.

**Definition 4:** [User Association Game] It is defined by the tuple  $\Gamma^\varpi = \{\mathcal{S}, \{X_i\}_{i \in \mathcal{S}}, \{U_i^\varpi\}_{i \in \mathcal{S}}\}$ ,  $\mathcal{S}$  is a set of BSs,  $X_i$  is a set of strategies of BS  $i$ , and  $U_i^\varpi$  is given in (12). Strategy set  $X_i$  is a discrete set of CRE bias values for small BS  $i \in \mathcal{B}_s$  and  $X_i$  is a discrete set of ABS ratios for macro BS  $i \in \mathcal{B}_e$ .

In the following, we first show the construction of  $N_i^\varpi$  and then in Proposition 2 we prove that the user association game  $\Gamma^\varpi$  is an  $\epsilon$ -potential game.

#### A. Base Station Neighborhood

We start with the definition of the neighbour set  $N_x$  of small BSs at location  $x$ :

$$N_x = \left\{ j \in \mathcal{S} \mid \max_{c_j} P_j g_j(x) c_j \geq \max_{k \in \mathcal{S}} \min_{c_k} P_k g_k(x) c_k \right\}. \quad (13)$$

BS  $j$  is in  $N_x$  if it is likely to serve the user at  $x$  for some CRE vector. Take the example of Fig. 2, which shows the bias received power range at a given location  $x$  for all BSs. The BSs whose biased received power range intersect with the line that passes through the max-min biased received power are the neighbour BSs. In Fig. 2, BS 1 is a macro BS and has a single possible CRE bias. It also has the max-min bias received power. This means a user at  $x$  will receive at least this bias power. The bias received power from BS 5 can exceed this max-min, so that BS 5 is likely to serve the user for some CRE bias and is thus included in  $N_x$ . In the same way, BS 7 is also included in  $N_x$ . On the other hand, the bias received power from BS 2 will never exceed that of BS 1 and thus BS 2 will never serve the user at  $x$ .

We now construct the neighbourhood of small BSs based on sets  $N_x$ . We assume that the users located at  $x$  calculate  $N_x$  and report it to their serving BS, which multicast this information to all the BSs in  $N_x$ . BS  $j$  is considered to be a neighbour of BS  $i$  if the proportion of reports where BS  $i$  and BS  $j$  are in  $N_x$  is at least a threshold  $\varpi$ . This constraint aims at excluding from the neighbourhood BSs that have insignificant influence on load. Otherwise, due to the infinite support of shadowing in our model, all BSs in the network can be potentially neighbours. Formally, the neighbour set of small BS  $i$  is defined as:

$$N_i^\varpi = \left\{ j \in \mathcal{S} \mid \frac{\int_{x \in \mathcal{L}} \lambda(x) 1_{i,j \in N_x} dx}{\int_{x \in \mathcal{L}} \lambda(x) 1_{i \in N_x} dx} \geq \varpi \right\}. \quad (14)$$

For the threshold  $\varpi = 0$  the neighbour set  $N_i^0$  boils down to  $N_i^0 = \bigcup_{x: i \in N_x} N_x$ . The neighbour set  $N_i^\varpi$  is empty for  $\varpi > 1$ . For  $0 < \varpi \leq 1$ ,  $N_i^\varpi$  is a decreasing sequence of sets.

Note that a change in the ABS ratio theoretically affects the load of all BSs of the network through interference. We thus assume that a macro BS neighborhood is made of all BSs. In practice however, the neighborhood of a macro BS is finite because interference power decreases with distance. Macro BS neighborhood can be constructed similarly to the small BS neighborhood construction as shown in Appendix H.

**Proposition 2:** The user association game  $\Gamma^\varpi$  with the potential function (8) is an  $\epsilon$ -potential game, where

$$\epsilon = \max_{a_i, a'_i \in X_i, a_{-i} \in X_{-i}, i \in \mathcal{S}} \left| \sum_{j \in N_i^0 \setminus N_i^\varpi} \frac{(1 - \rho_j(a_i, a_{-i}))^{1-\alpha}}{\alpha - 1} - \sum_{j \in N_i^0 \setminus N_i^\varpi} \frac{(1 - \rho_j(a'_i, a_{-i}))^{1-\alpha}}{\alpha - 1} \right|. \quad (15)$$

*Proof:* See Appendix D. ■

**Corollary 1:** The game  $\Gamma^0$  is an exact potential game.

## IV. DISTRIBUTED LEARNING ALGORITHMS

Recall that the potential function property enables finding a PNE through distributed learning algorithms. In this section, we introduce distributed learning algorithms that are used to find the optimal PNE of the user association game. First, we present the BR algorithm and the LLLA for the complete information setting. Next, the BLLLA for the partial information setting is described.

#### A. Best Response Algorithm

Best response algorithm is an asynchronous algorithm where at any given time only a single BS updates its strategy. Set  $\varpi$  and assume a time-varying random process with which a BS is chosen to revise its strategy<sup>3</sup>. The selected BS computes its cost  $U_i(a_i, a_{-i}(t-1))$  for all  $a_i \in X_i$  and sets  $U_i(a_i, a_{-i}(t-1)) = \infty$  if  $\rho_j \geq 1$  or  $O_j \geq \bar{O}_j$  for  $j \in N_i^\varpi$ . Then, the BS chooses a strategy  $a_i \in X_i$  that minimises its cost, given the strategies  $a_{-i} \in X_{-i}$  of other players. In other words, BS  $i$  chooses a strategy from its best response set  $B_i$ :

$$B_i(a_{-i}) = \arg \min_{a_i} U_i(a_i, a_{-i}). \quad (16)$$

Note that the BR algorithm requires complete information, i.e., the effects of choosing all the other strategies are supposed to be known. Moreover, BR algorithm is not guaranteed to converge to the optimal PNE even in exact potential game  $\Gamma^0$  because the potential function may have multiple sub-optima [44]. For the  $\epsilon$ -potential game  $\Gamma^\varpi$  ( $\varpi \neq 0$ ) and  $\epsilon \neq 0$ , a PNE may not even exist.

<sup>3</sup>Uniform probability or stationarity of the process is not required, it is only required that the probability of selecting any player is positive.

### B. Log-linear Learning Algorithm

The LLLA is a classical asynchronous algorithm that guarantees the convergence to the optimal PNE of an exact potential game [47]. This algorithm is similar to BR but allows deviations from the best response with a small probability. It is summarised in Algorithm 1. However, for this algorithm the BSs require again complete information. For example, given the strategies of others, the BS has to know the cost function value for all its strategies to compute (17). With this information, it selects a strategy to play according to a probability distribution. In general, acquiring this amount of information is not feasible. To overcome this difficulty in the next subsection we propose to use BLLLA.

---

#### Algorithm 1 Log-linear Learning Algorithm

---

- 1: **Initialisation:** Start with arbitrary action profile  $a$ .
- 2: Set parameter  $\tau$  and  $\varpi$ .
- 3: **While**  $t \geq 1$  **do**
- 4: Randomly select a player  $i$ .
- 5: Compute cost  $U_i(a_i, a_{-i}(t-1))$  for all  $a_i \in X_i$ .
- 6: For any  $a_i \in X_i$ , set  $U_i(a_i, a_{-i}(t-1)) = \infty$  if  $\rho_j \geq 1$  or  $O_j \geq \bar{O}_j$  for  $j \in N_i^\varpi$ .
- 7: Take action  $a_i(t)$  from  $X_i$  with probability  $p_i^{a_i}(t)$ ,

$$p_i^{a_i}(t) = \frac{\exp\left(-\frac{1}{\tau}U_i(a_i, a_{-i}(t-1))\right)}{\sum_{a'_i \in X_i} \exp\left(-\frac{1}{\tau}U_i(a'_i, a_{-i}(t-1))\right)}. \quad (17)$$

- 8: All the other players must repeat their previous actions, i.e.,  $a_{-i}(t) = a_{-i}(t-1)$ .
- 

### C. Binary Log-linear Learning Algorithm

The BLLLA converges to the optimal PNE of an exact potential game even if only partial information about the game is available to the players [47]. Partial information is the information that a player has about its current strategy. Unlike complete information the effect of choosing any other strategy is not known to the player. As LLLA, the BLLLA is also an asynchronous algorithm<sup>4</sup>. In this algorithm, whenever the BS updates its strategy it does it in two steps. In the first step, the BS tries a strategy from its strategy set to obtain its payoff. In the second step, the BS randomly chooses among the two strategies (present strategy and trial strategy) as summarised in Algorithm 2.

Note that in all the above algorithms the actions that are not in the feasible set  $\mathcal{F}$  have infinite cost. If there is no action in the feasible set  $\mathcal{F}$  at time  $t$  then by convention we set CRE bias equal to unity for small BSs and ABS ratio equal to zero for macro BSs.

#### D. Convergence of LLLA and BLLLA to Optimal PNE

The proof of convergence of LLLA and BLLLA to optimal PNE for an exact potential game  $\Gamma^0$  is given in [47]. Following a similar technique, we prove the convergence of LLLA and BLLLA to the global minimum of the potential function of a

<sup>4</sup>Asynchronicity is in fact only required within each neighborhood.

---

#### Algorithm 2 Binary Log-linear Learning Algorithm

---

- 1: **Initialisation:** Start with arbitrary action profile  $a$ .
- 2: Set parameter  $\tau$  and  $\varpi$ .
- 3: **While**  $t \geq 1$  **do**
- 4: Randomly select a BS  $i$ .
- 5: Select a trial action  $\hat{a}_i \in X_i$  with uniform probability.
- 6: Compute cost  $U_i(\hat{a}_i, a_{-i}(t-1))$ .
- 7: Set  $U_i(\hat{a}_i, a_{-i}(t-1)) = \infty$  if  $\rho_j \geq 1$  or  $O_j \geq \bar{O}_j$  for  $j \in N_i^\varpi$ .
- 8: Play action  $a_i(t) \in \{a_i(t-1), \hat{a}_i\}$  as given below.

$$a_i(t) = \begin{cases} a_i(t-1), & \text{w.p. } \frac{e^{-\frac{1}{\tau}U_i(a_i(t-1))}}{e^{-\frac{1}{\tau}U_i(a_i(t-1))} + e^{-\frac{1}{\tau}U_i(\hat{a}_i, a_{-i}(t-1))}} \\ \hat{a}_i, & \text{w.p. } \frac{e^{-\frac{1}{\tau}U_i(\hat{a}_i, a_{-i}(t-1))}}{e^{-\frac{1}{\tau}U_i(a_i(t-1))} + e^{-\frac{1}{\tau}U_i(\hat{a}_i, a_{-i}(t-1))}}. \end{cases} \quad (18)$$

- 9: All the other players must repeat their previous actions, i.e.,  $a_{-i}(t) = a_{-i}(t-1)$ .
- 

near-potential game  $\Gamma^\varpi$ ,  $\varpi > 0$ , in the following theorem. Unlike in [29], our proof does not assume the underlying Markov chain to be reversible, which allows to extend their result to the cases where BSs are chosen in a non-stationary and state-dependent way to revise their actions. As the underlying Markov chain is ergodic, each state has a positive probability to be chosen throughout the iterations of the algorithm. We say that the algorithm *converges* to a state if that probability is non zero when parameter  $\tau$  goes to zero.<sup>5</sup>

Let  $\phi_\alpha^*$  and  $\phi_\alpha^\dagger$  be the first minimum and second minimum values of the potential.

**Theorem 3:** Let  $\xi = \max_{a,b \in X, i \in \mathcal{S}} |\xi_i^\varpi(a,b)|$ , where  $\xi_i^\varpi(a,b)$  is given in (35) with action profiles  $a$  and  $b$  that differs at most in component  $i$ . For any  $\epsilon > 0$  and any  $\xi$ -potential game  $\Gamma^\varpi$  with  $\xi < \frac{\epsilon}{2(|X|-1)}$ , LLLA and BLLLA converge to a set of  $\xi$ -NEs with potential less than  $\phi_\alpha^* + \epsilon$ .

*Proof:* See Appendix E. ■

**Corollary 2:** For the game  $\Gamma^\varpi$ , if  $\xi < \frac{\phi_\alpha^\dagger - \phi_\alpha^*}{2(|X|-1)}$  then both the LLLA and BLLLA converge to a set of PNEs whose potential value is  $\phi_\alpha^*$ .

*Proof:* See Appendix F. ■

We now define the neighborhood of every BS so that the condition of Corollary 2 is met. The following theorem gives an upper bound on  $\varpi$  that guarantees LLLA and BLLLA to converge to an optimal PNE.

**Theorem 4:** The constraint in Theorem 3 is satisfied if

$$\varpi \leq \epsilon Q (1 - \rho_{\max})^\alpha, \quad (19)$$

where  $\rho_{\max}$  is the maximum possible load of a BS,  $Q = \frac{\max_{x, \theta, j \in \mathcal{S}} \frac{1}{v_j(x, \theta)}}{4|\mathcal{S}||X|\lambda_m \max_x \left\{ \frac{1}{\mu(x)} \right\}}$ , and  $\lambda_m$  is an upper bound for the sum arrival rate in a cell.

*Proof:* See Appendix G. ■

**Corollary 3:** The constraint in corollary 2 is satisfied if

$$\varpi \leq Q (\phi_\alpha^\dagger - \phi_\alpha^*) (1 - \rho_{\max})^\alpha. \quad (20)$$

**Remark** Recall that we have assumed that the neighborhood of every macro BS is made of all the BSs in the network. It is

<sup>5</sup>Or, say otherwise, that the state is *stochastically stable*, see, e.g., [47].

TABLE I: Simulation parameters.

Parameter	Variable	Value
Number of BSs	$N_s$	8
Macro BS during NS	$P_{\text{macro}}$	46 dBm
Macro BS during ABS	$P_{\text{ABS}}$	0 dBm
Small BS	$P_{\text{small}}$	24 dBm
Average file size	$\frac{1}{\mu}$	0.5 Mbytes
Average traffic load density	$\frac{\lambda}{\mu}$	64 bits/s/m <sup>2</sup>
System bandwidth	$W$	20 MHz
Carrier frequency	$f_c$	2.6 GHz
Noise power	$N$	-174+10log(W) dBm
Minimum SINR	$\gamma_{\text{min}}$	-10 dB
Path-loss exponent	$\eta$	3.5
Reference distance	$d_0$	10 m
CRE bias set	$c_i$	{1, 1.1, 1.2, ..., 16}
ABS ratio	$\theta_i$	{0, 0.01, 0.02, ..., 1}

however also possible to restrict it by following a similar technique as for small BSs. The definition (14) should be extended by considering the set of macro BSs that significantly affect the load of BS  $i$ . Then, similar conditions for convergence of LLLA and BLLLA can be obtained. See Appendix H for more details.

#### E. Effect of time-varying neighbours

For all the above algorithms, the BS  $i$  needs to know its neighbours  $N_i^\varpi$  to calculate its cost. Providing to every BS the neighbours set is a standard task for network operators for a given CRE bias vector. This task can be performed automatically e.g., using automated neighbour relation (ANR) standardised by 3GPP [48]. The difficulty here is to determine  $N_i^\varpi$  that depends on all the possible values of the CRE bias vector (see (14)). To address this problem, we propose the following technique.

At every packet call the user at location  $x$  calculates  $N_x$  and reports it to its serving BS, which multicast this information to all the BSs in  $N_x$ . In the process of learning, BS  $i$  updates  $N_i^0$  whenever it receives reports from other BSs. From this the BS calculates the proportion of reports of neighbour BSs. If the proportion of reports of BS  $j$  exceeds the threshold  $\varpi$  then it is included in  $N_i^\varpi$ . Since, the shadowing and the traffic is stationary the estimate of proportion of reports converges over time. Hence,  $N_i^\varpi \forall i$ , also converges. Thus, the algorithms LLLA and BLLLA converge to the global minimum of the objective function as proved in Corollary 2.

## V. SIMULATION RESULTS

In this section, we show simulation results considering standard parameters as adopted in 3GPP [49]. These parameters are listed in the Table I. We consider 8 BSs located in a two dimensional region  $\mathcal{L}$ . BS 1 is a macro BS that transmits with  $P_{\text{macro}}$  and the rest are small BSs that transmit with  $P_{\text{small}}$ . The user traffic varies with location across an average traffic density of 64 bits/s/m<sup>2</sup>. There are two hotspots where the traffic is 5 times the average traffic, which can be seen in Fig. 3. We consider shadow fading with a standard deviation of  $\sigma_{sh} = 8$  dB and a decorrelation distance of  $D_c = 20$  m. Cross correlation between the shadowing components at a location is considered to be 0.5. We use the classical Shannon formula for calculating channel capacity  $\tilde{\nu}_i^f(x, \bar{\theta}) = W \log_2 \left( 1 + \gamma_i^f(x, \bar{\theta}) \right)$ .

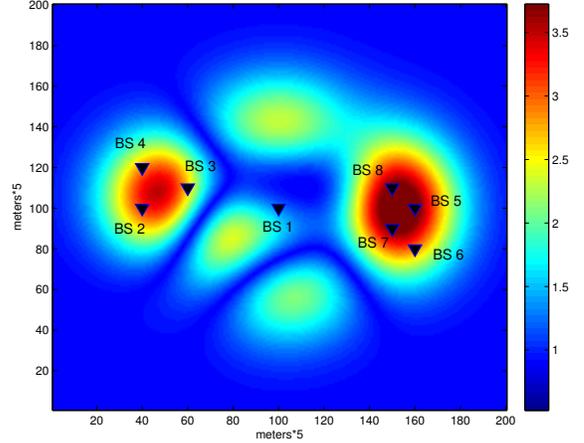


Fig. 3: Normalised traffic variations.

#### A. Performance Comparison of Algorithms

We first focus on the CRE optimisation and assume in this section that the macro BS does not implement ABS ( $\theta_1 = 0$ ). We consider a square region  $\mathcal{L}$  of side 1000 m.

The convergence of LLLA and BLLLA to the global minimum of the objective function is shown in Fig. 5 for  $\tau = 10^{-3}$ ,  $\varpi = 10^{-22}$ , and different values of  $\alpha$ . We observe that in all cases proposed algorithms converge within few tens and sometimes few hundreds of iterations. We also see that LLLA converges faster than BLLLA. This is of course due to the complete information setting assumed for LLLA. Note however that BLLLA does not loose so much in terms of convergence speed; this is an interesting conclusion for a practical implementation.

We also compare LLLA and BLLLA with Pradelski and Young learning algorithm (PYLA), which is completely uncoupled and a variant of trial and error algorithm [31]. It guarantees asymptotic convergence to the optimal NE for any finite game that possess at least one NE. Hence, it is a suitable algorithm to compare with LLLA and BLLLA for near-potential game  $\Gamma^\varpi$ . The comparison of evolution of objective function  $\phi_{50}$  is shown in Fig. 6<sup>6</sup>. LLLA and BLLLA converge quickly whereas PYLA oscillates between fast search and slow search phases. Therefore, the performance of LLLA and BLLLA for load balancing is much superior to that of PYLA.

1) *Effect of  $\varpi$* : The effect of threshold  $\varpi$  on the convergence of LLLA is shown in Fig. 7a for  $\tau = 10^{-3}$  and for the particular case  $\alpha = 50$  (results for BLLLA and other values of  $\alpha$  provide similar conclusions). For  $\varpi = 0$  all the BSs are neighbours so that our framework is an exact potential game and LLLA converges to an optimal PNE. Threshold parameter  $\varpi = 10^{-22}$  results in an  $\epsilon$ -potential game and satisfies the sufficient condition of Corollary 3. Therefore, LLLA converges also to the global minimizer of the objective function. Although small, this value of  $\varpi$  significantly shrinks

<sup>6</sup>The exploration parameter  $\epsilon$  of PYLA is set to  $\epsilon = 0.1$ . The functions that govern slow and fast search phases PYLA are chosen to satisfy the conditions [31, (1), (2), and (3)].

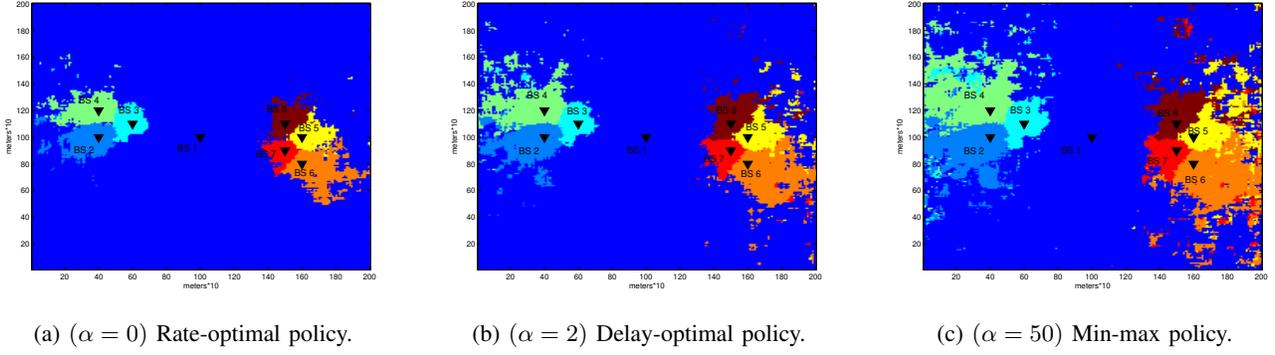


Fig. 4: The variations of the coverage regions of BSs obtained using the optimal CRE for different  $\alpha$ .

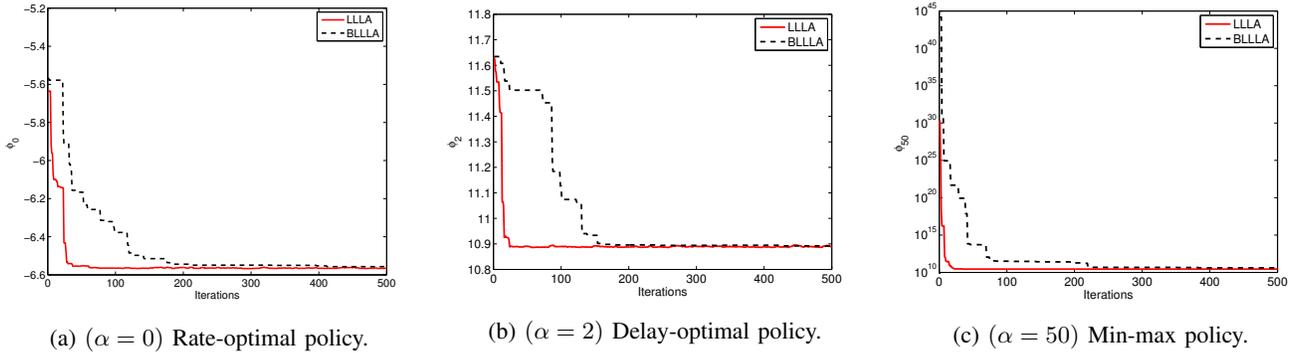


Fig. 5: Convergence of LLLA and BLLLA ( $\tau = 0.001$ ,  $\varpi = 10^{-22}$ ).

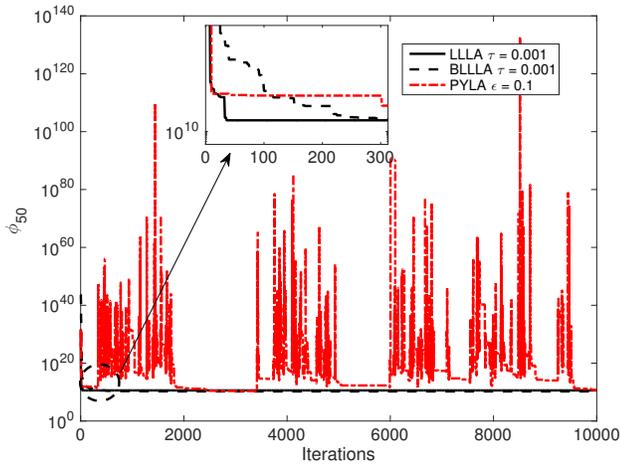


Fig. 6: Comparison of LLLA, BLLLA, and PYLA for  $\alpha = 50$ ,  $\varpi = 10^{-22}$ , and  $\epsilon = 0.1$ .

the neighborhood set of the BSs. In the scenario of Fig. 3, BSs 2 and 4 are for example not anymore neighbours of BSs 5, 6, and 7. If we now further increase  $\varpi$  to a value that violates the condition of Corollary 3 ( $\varpi = 0.9$ ), neighborhoods are further reduced. In our scenario, BS 1 is for example the only neighbour of BSs 3, 6, and 8. LLLA is however not anymore guaranteed to converge to an optimal PNE as seen from the figure. The threshold  $\varpi$  therefore strikes a balance between the size of the neighborhood and the optimality of the solution.

2) *Effect of  $\tau$* : There is a trade-off in the choice of  $\tau$ . LLLA and BLLLA converge with high probability to the global minimum of the objective function for  $\tau \in (0, \infty)$  under the conditions of Corollary 2. This means that, asymptotically, the probability that the algorithm is at the global minimum approaches to one as  $\tau$  goes to zero.

For high values of  $\tau$ , LLLA and BLLLA results into oscillations. This is due to the fact that the algorithms converge fastly in probability to the uniform distribution. As a matter of fact, it doesn't spend much time in optimal states, which is not practically desirable. For small values of  $\tau$ , asymptotically, the algorithms will spend most of time in the global optimal. However, convergence is slow in probability. This explains that the system can take long time to escape from sub-optimal states. Contrary to best response however, the proposed algorithms will not get stuck into these sub-optimal states.

The effect of  $\tau$  on the convergence of LLLA and BLLLA for  $\alpha = 2$  and  $\varpi = 10^{-22}$  is shown in Fig. 7b and Fig. 7c, for high value of  $\tau = 0.05$ , LLLA results into oscillations. It reaches the global minimum (around iteration 300) but does not spend much time in the optimum states. For  $\tau = 0.01$ , the time spent in the optimum states is increased. For a carefully chosen  $\tau = 0.001$ , the system spends most of the time in the optimal states. We can see in Fig. 7c that the behavior of BLLLA is similar to that of LLLA, except that it takes more iterations to hit the global minimum of the potential function because of the partial information used.

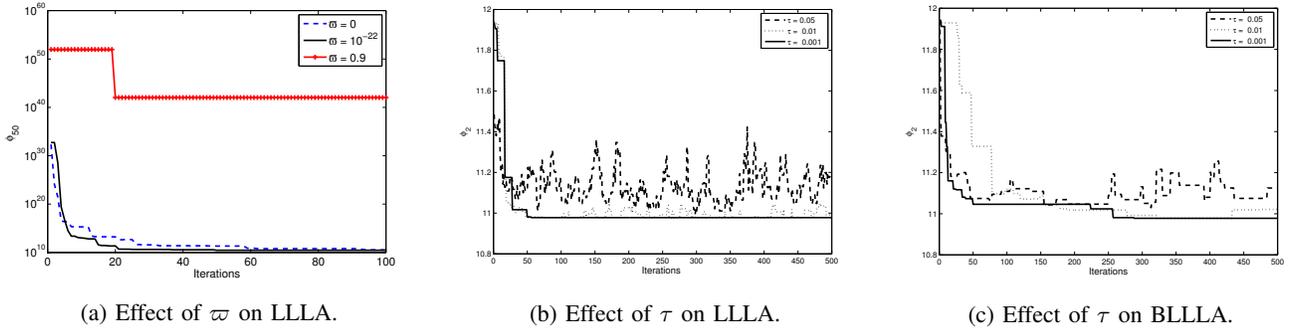


Fig. 7: Effect of  $\varpi$  and  $\tau$  on the convergence of LLLA and BLLA.

TABLE II: Comparison of optimal CRE, optimal loads of BSs for different  $\alpha$  ( $\tau = 10^{-3}$ ,  $\varpi = 10^{-22}$ ).

BS $i$	$\alpha = 0$		$\alpha = 2$		$\alpha \rightarrow \infty$	
	$c_i^*$	$\rho_i^* \%$	$c_i^*$	$\rho_i^* \%$	$c_i^*$	$\rho_i^* \%$
1	1	92	1	61	1	45
2	1	7	3	20	8	42
3	1	4	3	9	9	23
4	1	9	3	18	8	37
5	1	11	3	21	7	37
6	1	8	3	20	7	43
7	1	5	3	11	8	30
8	1	7	3	19	6	37

3) *Effect of  $\alpha$* : We now compare in Table II the optimal bias values and BS loads. Corresponding coverage regions are shown in Fig. 4. With  $\alpha = 0$ , every user is served by the BS that provides maximum data rate, which is obtained for bias values equal to one. This corresponds to the classical best signal association rule that results in heavy load imbalance between stations: the load of the macro BS reaches 92%, while small BSs have loads less than 11%. As  $\alpha$  increases to 2 the coverage regions of all small BSs expands and that of the macro BS shrinks. The load of the macro BS is decreased to 61% and concurrently the utilisation of small BSs is increased (up to 21%). Min-max policy is approximated with a value of  $\alpha = 50$ . The load of the macro BS is further reduced to 45% and load dispersion is decreased.

This phenomenon can also be observed in figures 8 and 9, where optimal CRE and loads are shown as functions of  $\alpha$ . Every point on this figure is obtained by averaging over 50 realisations of LLLA. By definition, the CRE of the macro BS is constant and equal to 1. We see on this figure how the CREs of small BSs are gradually increased and how the load dispersion is reduced. We also conclude that choosing  $\alpha = 50$  provides a good approximation for the min-max policy in this scenario.

### B. Fairness-Outage Tradeoff Using ABS

In this section, we allow the macro BS to implement ABS and study the effect of this technique on outage and fairness. For the sake of simplicity, we set  $\tau = 10^{-3}$ ,  $\varpi = 10^{-22}$  and use LLLA. We also consider a square region  $\mathcal{L}$  of side 2000 m in the simulations. Three cases may be compared to evaluate the interest of using ABS:

- 1) *No outage constraint no ABS*: In this case, the macro BS does not implement ABS ( $\theta_1 = 0$ ) and we do not

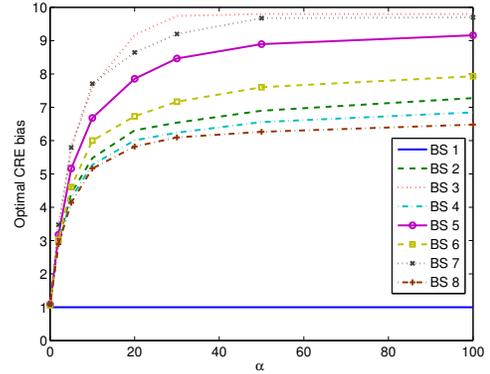


Fig. 8: Evolution of optimal CRE bias.

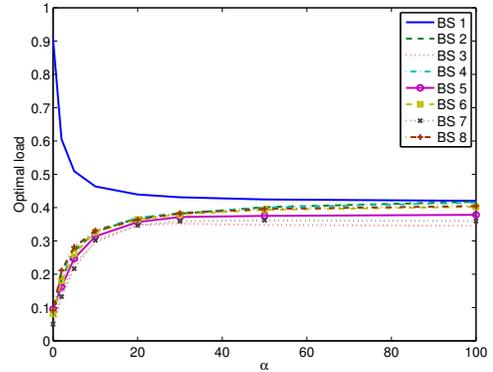


Fig. 9: Evolution of optimal loads.

impose outage constraint. This serves as a benchmark to compare with other two cases.

- 2) *Outage constraint without ABS*: We introduce here an outage constraint ( $\bar{O}_i = 2\%$  for all  $i$ ) but still don't allow ABS ( $\theta_1 = 0$ ). Outage is taken into account by setting an infinite cost to actions violating the constraint  $\bar{O}_i$  as shown in Algorithm 1.
- 3) *Outage constraint with ABS*: We impose an outage constraint and allow ABS at the macro BS.

In Fig. 10, we show the cost function  $\phi_\alpha$  for the three considered cases and different  $\alpha$ . Each curve of the figure is obtained by averaging over 50 realisations. In our scenario, choosing  $\alpha = 0$  (Fig. 10a) results in a low outage probability. Therefore,  $\phi_\alpha$  curves for the first two cases are very close

to each other. Since outage constraint is already met without ABS, introducing ABS does not bring any additional interest. With  $\alpha = 2$  (Fig. 10b), outage probability increases but remains below the acceptable threshold. As a consequence, the three curves also converge to the same value. With  $\alpha = 50$  (Fig. 10c), outage probabilities exceed the threshold in the first case. When outage constraint is introduced in the second case, the cost increases because the feasible set shrinks (some actions are not anymore available). When ABS is introduced in the third case, the feasible set expands again and so the optimal cost of the system decreases, fairness is improved.

Fig. 11 shows the evolution of the outage probabilities as the algorithm iterates (we assume  $\alpha = 50$  and 50 realisations are averaged). With no ABS and no constraint (Fig. 11 (i)), outage probabilities of BS 3 and 7 considerably exceed the threshold of 2%. The reason is that small BSs increase their CRE to achieve optimality without taking care of users in outage, so that users at cell edge may experience a very bad signal quality. Therefore, fairness is achieved at the cost of an unacceptable outage. Imposing an outage constraint without using ABS is sufficient to achieve a good quality of service (Fig. 11 (ii)). The function  $\phi_\alpha$  however converges to a higher value (see Fig. 10c). ABS is a good means to both meet the outage constraint and achieve fairness (Fig. 11 (iii)). The reason is that small BSs cell edge users experience a better signal quality during ABS subframes and the ABS ratio offers also the macro BS an additional degree of freedom for adapting its load and achieving fairness. This can also be seen in Fig. 12, where we have plotted average loads over 50 realisations after LLLA has converged. From the first to the second case, the load vector expands because of the smaller feasible set and then shrinks in the third case thanks to ABS.

## VI. CONCLUSIONS

In this paper, a novel approach for load balancing using CRE association technique and ABS interference management technique is presented. Our approach exploits the near-potential game structure and distributed learning algorithms. We showed that unlike in the literature the load balancing problem can be solved distributively by restricting the number of neighbours. We provide extensive proofs of convergence of learning algorithms. We also provide sufficient conditions under which learning algorithms converges to the optimal PNE. By running extensive simulations in two settings, which are complete and partial information settings, we show that the proposed algorithms converge within a few tens of iterations to the optimal PNE, which is also a minimiser of a  $\alpha$ -fairness function of the network. The convergence speed of the BLLL that uses partial information is comparable to the LLLA that uses complete information, meaning that partial information is sufficient in practical implementations. Simulations showed that for load balancing LLLA and BLLL perform better than a variant of trial and error algorithm. Finally, we showed that by introducing ABS the outages can be reduced and a better load balancing can be achieved.

## APPENDIX

### A. Proof of Rate-optimal Policy

*Proof:* Minimising  $\phi_0(\bar{c}, \bar{\theta})$  is equivalent to minimise the arithmetic mean of the BSs loads, i.e.,  $\sum_{i \in |S|} \rho_i(\bar{c}, \bar{\theta})$ . We now prove that minimising  $\phi_0(\bar{c}, 0)$  is indeed rate-optimal policy by contradiction. Consider that at the minimum  $\phi_0^*$  a location  $x_0$  is associated with a BS  $j$  with rate  $r_j(x_0)$ , which is not the maximum. Then, there exists a BS  $k$  that provides the maximum rate  $r_k(x_0)$ . Let  $\phi_0^k$  be the value of the objective function when the UE at location  $x_0$  is associated with BS  $k$ . The difference of the objective function due to the loads of BSs  $j$  and  $k$  is

$$\phi_0^k - \phi_0^* = \frac{\lambda(x_0)}{\mu(x_0)} \left( \frac{1}{r_k(x_0)} - \frac{1}{r_j(x_0)} \right) < 0, \quad (21)$$

which is a contradiction that  $\phi_0^*$  is the minimum. Therefore, we conclude that at  $\phi_0^*$  all the locations will be served by the BS that provide the highest rate. ■

### B. Proof of Theorem 1

The proof is divided into the following two lemmas. The first lemma 2 gives the property that is required for proving the next lemma 3, which concludes the proof of the theorem 1.

**Lemma 2:** Let  $f_\alpha(r) = \sum_{i \in S} \frac{r_i^{1-\alpha}}{1-\alpha}$ . If  $r \succ y$  then there is  $A > 0$  large enough such that for all  $\alpha \geq A$ ,  $f_\alpha(r) > f_\alpha(y)$ .

*Proof:* Let  $\alpha > 1$ . Without loss of generality, assume that  $r$  and  $y$  are sorted in increasing order and that  $r_1 > y_1$ . Let  $\delta = r_1 - y_1$ . Then

$$(1-\alpha)(f_\alpha(r) - f_\alpha(y)) = \sum_{i=1 \dots n} (r_i^{1-\alpha} - y_i^{1-\alpha}) \quad (22)$$

$$\leq r_1^{1-\alpha} - y_1^{1-\alpha} + \sum_{i=2 \dots n} (r_i^{1-\alpha} - y_i^{1-\alpha}) \quad (23)$$

$$\leq r_1^{1-\alpha} - (r_1 - \delta)^{1-\alpha} + (n-1)r_1^{1-\alpha} - \sum_{i=2 \dots n} y_i^{1-\alpha} \quad (24)$$

$$\leq nr_1^{1-\alpha} - (r_1 - \delta)^{1-\alpha}. \quad (25)$$

Then we have  $f_\alpha(r) > f_\alpha(y)$  if and only if

$$nr_1^{1-\alpha} - (r_1 - \delta)^{1-\alpha} \leq 0 \quad (26)$$

$$\Leftrightarrow \frac{\log(n)}{\log\left(\frac{r_1 - \delta}{r_1}\right)} \geq 1 - \alpha \quad (27)$$

$$\Leftrightarrow 1 + \frac{\log(n)}{\log(r_1) - \log(r_1 - \delta)} \leq \alpha. \quad (28)$$

**Lemma 3:** Let  $\mathcal{X}$  be a compact subset of  $\mathbb{R}^{|S|}$ . Consider the set

$$Z = \bigcap_{A > 1} \bigcup_{\alpha \geq A} \overline{\operatorname{argmax}_{x \in \mathcal{X}} f_\alpha(x)}.$$

Then  $Z$  is non-empty and is made of max-min vectors in  $\mathcal{X}$ .

*Proof:* Let  $Z_A = \overline{\bigcup_{\alpha \geq A} \operatorname{argmax}_{x \in \mathcal{X}} f_\alpha(x)}$ . It is a decreasing nested sequence of non-empty compact sets. By Cantor's intersection theorem, it is not empty and compact.

Let  $x^* \in \bigcap_{A > 1} Z_A$ . There is an increasing sequence  $\alpha(n)$  and  $x_{\alpha(n)} \in \operatorname{argmax}_{x \in \mathcal{X}} f_{\alpha(n)}(x)$  with  $x_{\alpha(n)} \rightarrow x^*$ . Assume there

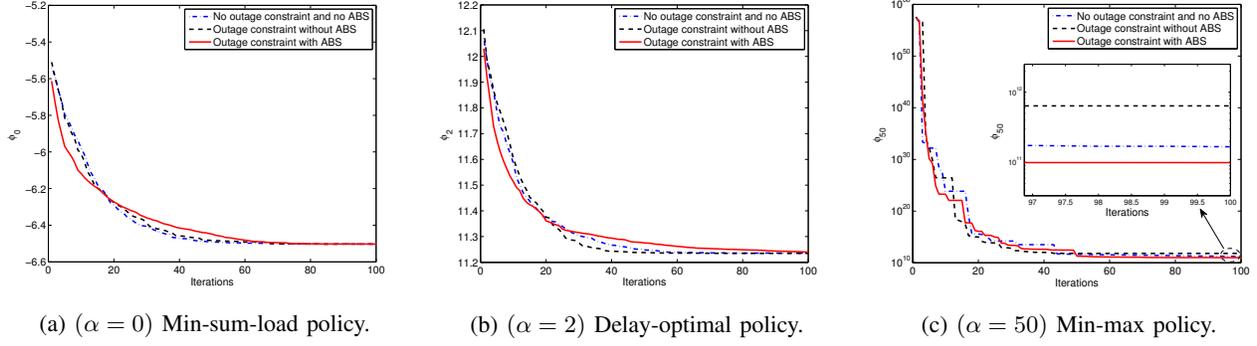


Fig. 10: Effect of outage constraint and ABS (LLLA,  $\tau = 10^{-3}$ ,  $\varpi = 10^{-22}$ ,  $\bar{O}_i = 2\%$ ).

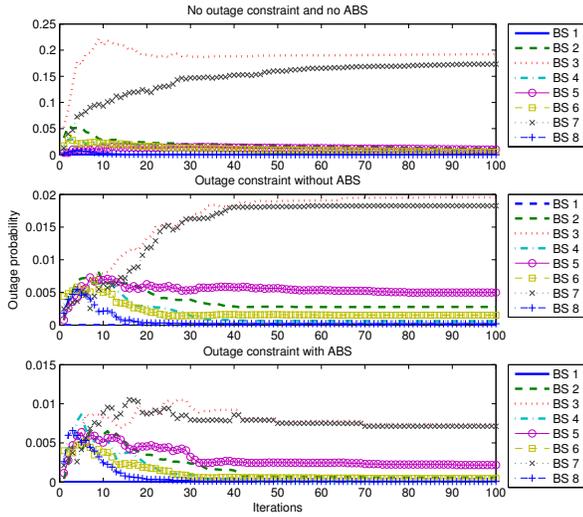


Fig. 11: Outage probability comparisons of (i) no outage constraint no ABS, (ii) outage constraint without ABS, and (iii) outage constraint with ABS (LLLA,  $\alpha = 50$ ,  $\tau = 10^{-3}$ ,  $\varpi = 10^{-22}$ ,  $\bar{O}_i = 2\%$ ).

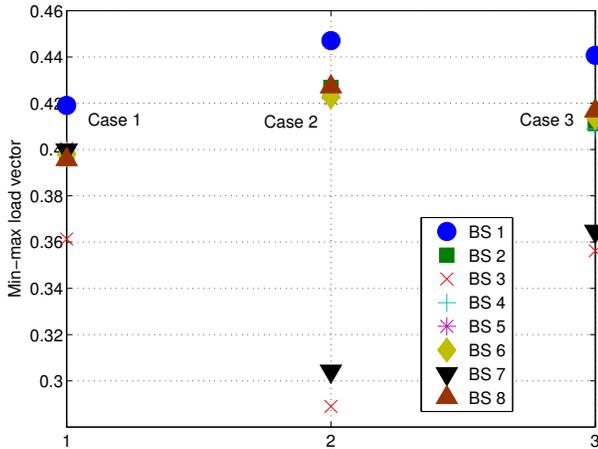


Fig. 12: Comparison of min-max load vector of cases (i) no outage constraint no ABS, (ii) outage constraint without ABS, and (iii) outage constraint with ABS (LLLA,  $\alpha = 50$ ,  $\tau = 10^{-3}$ ,  $\varpi = 10^{-22}$ ,  $\bar{O}_i = 2\%$ ).

is  $y \succ x^*$ . Then, by Lemma 2, there is  $N$  such that, for every  $n \geq N$ ,  $f_{\alpha(n)}(y) > f_{\alpha(n)}(x^*)$ . But, by definition of  $x_{\alpha(n)}$ ,  $f_{\alpha(n)}(x_{\alpha(n)}) \geq f_{\alpha(n)}(y)$ , which is a contradiction with the fact that  $f_{\alpha(n)}(x_{\alpha(n)}) \rightarrow f_{\alpha(n)}(x^*)$ , which is ensured by Berge maximum theorem. ■

### C. Proof of Lemma 1

*Proof:* Let  $h$  be a potential functions of both the games. Since,  $a^*$  is a PNE for  $\mathcal{G}$ , then for all players  $i$  and  $a_i$  we have:

$$U_i(a^*) - U_i(a_i, a_{-i}^*) \leq 0, \quad (29)$$

$$h(a^*) - h(a_i, a_{-i}^*) \leq 0. \quad (30)$$

The second inequality follows from Definition 3. Then for  $\mathcal{G}'$ , we have:

$$\begin{aligned} U'_i(a^*) - U'_i(a_i, a_{-i}^*) &\leq U'_i(a^*) - U'_i(a_i, a_{-i}^*) + h(a_i, a_{-i}^*) - h(a^*) \\ &\leq |U'_i(a^*) - U'_i(a_i, a_{-i}^*) + h(a_i, a_{-i}^*) - h(a^*)| \leq \epsilon. \end{aligned} \quad (31)$$

First inequality is obtained by adding a positive term and the third inequality is by Definition 3. It follows from the third inequality that  $a^*$  is an  $\epsilon$ -NE of  $\mathcal{G}'$ . ■

### D. Proof of Proposition 2

*Proof:* We prove that the game  $\Gamma^\varpi$  is an  $\epsilon$ -potential game by verifying the definition 3 for the two BS types (macro or small). Consider action profiles  $a = (a_i, a_{-i})$  and  $b = (a'_i, a_{-i})$ . First, consider a macro BS  $i \in \mathcal{B}_e$ . The neighbourhood of BS  $i$  includes all the BSs in the network, i.e.,  $N_i^\varpi = N_i^0 = \mathcal{S}$ . With this, we have

$$U_i^\varpi(a) - U_i^\varpi(b) = \phi_\alpha(a) - \phi_\alpha(b).$$

Therefore, inequality in the definition 3 holds for all macro BSs.

Next, we show that it also holds for any small BS. Consider small BS  $i \in \mathcal{B}_s$ . The neighborhood of small BS  $i$  is given by (14). From the objective function (8) we have

$$\phi_\alpha(a) - \phi_\alpha(b) = \sum_{j \in \mathcal{S}} \frac{(1 - \rho_j(a))^{1-\alpha}}{\alpha - 1} - \sum_{j \in \mathcal{S}} \frac{(1 - \rho_j(b))^{1-\alpha}}{\alpha - 1}, \quad (32)$$

Since, the action of BS  $i$  only affects its neighbour BSs we have

$$\phi_\alpha(a) - \phi_\alpha(b) = \sum_{j \in N_i^0} \frac{(1 - \rho_j(a))^{1-\alpha}}{\alpha - 1} - \sum_{j \in N_i^0} \frac{(1 - \rho_j(b))^{1-\alpha}}{\alpha - 1}. \quad (33)$$

It can be splitted as below.

$$\begin{aligned} \phi_\alpha(a) - \phi_\alpha(b) &= \sum_{j \in N_i^\varpi} \frac{(1 - \rho_j(a))^{1-\alpha}}{\alpha - 1} + \sum_{j \in N_i^0 \setminus N_i^\varpi} \frac{(1 - \rho_j(a))^{1-\alpha}}{\alpha - 1} \\ &\quad - \sum_{j \in N_i^\varpi} \frac{(1 - \rho_j(b))^{1-\alpha}}{\alpha - 1} - \sum_{j \in N_i^0 \setminus N_i^\varpi} \frac{(1 - \rho_j(b))^{1-\alpha}}{\alpha - 1}. \end{aligned} \quad (34)$$

Let denote

$$\xi_i^\varpi(a, b) = \sum_{j \in N_i^0 \setminus N_i^\varpi} \frac{(1 - \rho_j(a))^{1-\alpha}}{\alpha - 1} - \sum_{j \in N_i^0 \setminus N_i^\varpi} \frac{(1 - \rho_j(b))^{1-\alpha}}{\alpha - 1}. \quad (35)$$

Then using (12) the above equation can be written as below.

$$\phi_\alpha(a) - \phi_\alpha(b) = U_i^\varpi(a) - U_i^\varpi(b) + \xi_i^\varpi(a, b). \quad (36)$$

Then we have

$$|U_i^\varpi(a) - U_i^\varpi(b) + \phi_\alpha(b) - \phi_\alpha(a)| \leq \max_{a, b \in X, i \in S} |\xi_i^\varpi(a, b)|. \quad (37)$$

Hence, the inequality in the definition 3 also holds for all small BSs. Therefore,  $\Gamma^\varpi$  is an  $\epsilon$ -potential game. ■

### E. Proof of Theorem 3

*Proof:* The proof is given for LLLA only. Whereas, for BLLLA it can be done following the similar approach. Note that for the game  $\Gamma^\varpi$  both the LLLA and BLLLA induces a regular perturbed Markov process over the action space  $X = X_1 \times X_2 \times \dots \times X_{|S|}$ .

The resistance  $R(a^0 \rightarrow a^1)$  of any feasible transition  $a^0 = (a_i^0, a_{-i}^0) \rightarrow a^1 = (a_i^1, a_{-i}^1)$  for the LLLA is given as [47]:

$$R(a^0 \rightarrow a^1) = \phi_\alpha(a^1) - \phi_\alpha(B_i(a_{-i}^0), a_{-i}^0) + \xi_i^\varpi((B_i(a_{-i}^0), a_{-i}^0), a^1), \quad (38)$$

where  $B_i(a_{-i}^0)$  is the set of best response actions.

Let  $\phi_\alpha^*$  be the global minimum value of the potential. Let  $a^*$  be a state with potential  $\phi_\alpha^*$ . Action profile  $a^*$  is an optimal PNE of  $\Gamma^0$  and a  $\xi$ -NE of  $\Gamma^\varpi$  according to Lemma 1.

Suppose that a minimum resistance tree,  $\mathcal{T}$ , is rooted at an action profile  $a$  whose potential is more than  $\phi_\alpha^* + \epsilon$ . This means the action profile  $a$  is such that

$$\phi_\alpha(a^*) - \phi_\alpha(a) \leq -\epsilon. \quad (39)$$

Since  $\mathcal{T}$  is a rooted tree, there exists a path  $\mathcal{P}$  from  $a^*$  to  $a$  of the form:

$$\mathcal{P} = \{a^* \rightarrow a^1 \rightarrow \dots \rightarrow a^m \rightarrow a\}. \quad (40)$$

Consider the reverse path  $\mathcal{P}^r = \{a \rightarrow a^m \rightarrow \dots \rightarrow a^1 \rightarrow a^*\}$ . The difference in the

resistance of the paths can be calculated using (38) and can shown to be as given below.

$$R(\mathcal{P}) - R(\mathcal{P}^r) = \phi_\alpha(a) - \phi_\alpha(a^*) + \sum_{a, b \in \mathcal{P}, j \in S} \xi_j^\varpi(a, b) - \sum_{c, d \in \mathcal{P}^r, j \in S} \xi_j^\varpi(c, d). \quad (41)$$

Construct a new tree  $\mathcal{T}^1$  rooted at  $a^*$  by adding the edges of  $\mathcal{P}^r$  to  $\mathcal{T}$  and removing the redundant edges  $\mathcal{P}$ . The new tree will have the following resistance:

$$R(\mathcal{T}^1) = R(\mathcal{T}) + R(\mathcal{P}^r) - R(\mathcal{P}), \quad (42)$$

$$= R(\mathcal{T}) + \phi_\alpha(a^*) - \phi_\alpha(a)$$

$$+ \sum_{a, b \in \mathcal{P}^r, j \in S} \xi_j^\varpi(a, b) - \sum_{c, d \in \mathcal{P}, j \in S} \xi_j^\varpi(c, d), \quad (43)$$

$$< R(\mathcal{T}) + \phi_\alpha(a^*) - \phi_\alpha(a) + 2 \sum_{a, b \in \mathcal{P}^r, j \in S} \xi, \quad (44)$$

$$< R(\mathcal{T}) + \phi_\alpha(a^*) - \phi_\alpha(a) + 2(|X| - 1)\xi, \quad (45)$$

$$\leq R(\mathcal{T}) - \epsilon + 2(|X| - 1)\xi, \quad (46)$$

where  $(|X| - 1)$  is the total number of edges in the tree  $\mathcal{T}^1$ . We have  $R(\mathcal{T}^1) < R(\mathcal{T})$  if  $\xi < \frac{\epsilon}{2(|X| - 1)}$ . Therefore, we can construct the tree  $\mathcal{T}^1$  rooted at  $a^*$  with strictly less resistance than the tree  $\mathcal{T}$  leading to a contradiction that  $\mathcal{T}$  is minimal resistance tree. ■

### F. Proof of Corollary 2

*Proof:* By substituting  $\epsilon = \phi_\alpha^\dagger - \phi_\alpha^*$  in Theorem 3, we obtain that the algorithm converges to a set of states with potential strictly less than  $\phi_\alpha^* + (\phi_\alpha^\dagger - \phi_\alpha^*) = \phi_\alpha^\dagger$ . Hence, the only possible states are those with potential value  $\phi_\alpha^*$ . Let  $a^*$  be one such state, and assume that it is not a PNE of  $\Gamma^\varpi$ . Then there exists a BS  $i$  and an action  $a_i$ , such that:  $U_i^\varpi(a^*) - U_i^\varpi(a_i, a_{-i}^*) > 0$ . Since  $\phi_\alpha(a_i, a_{-i}^*) - \phi_\alpha(a^*) \geq \phi_\alpha^\dagger - \phi_\alpha^* = \epsilon$  it follows  $|U_i^\varpi(a^*) - U_i^\varpi(a_i, a_{-i}^*) + \phi_\alpha(a_i, a_{-i}^*) - \phi_\alpha(a^*)| > \epsilon$  which is a contradiction with the game  $\Gamma^\varpi$  being a  $\xi$ -potential game with  $\xi < \frac{\epsilon}{2|X|}$ . ■

### G. Proof of Theorem 4

*Proof:* Let  $a = (a_i, a_{-i})$  and  $b = (a'_i, a_{-i})$  be two action profiles, where BS  $i$  changes its action. Let  $\varrho_i(x, a) = \frac{\lambda(x)}{\mu(x)\nu_j(x, a)}$ . Let denote  $\nu_m = \max_{x, \bar{\theta}, j \in S} \frac{1}{\nu_j(x, \bar{\theta})}$ ,  $\mu_m = \max_x \frac{1}{\mu(x)}$ , and  $\lambda_m = \max_{a \in X, j \in S} \int_{x \in D_j(a)} \lambda(x) dx$ .

The parameter  $\varpi$  is such that (14):

$$j \in N_i^0 \setminus N_i^\varpi \Rightarrow 0 < \frac{\int_x \lambda(x) \mathbf{1}_{i, j \in N_x} dx}{\int_x \lambda(x) \mathbf{1}_{i \in N_x} dx} < \varpi.$$

The change in load of BS  $j$  due to change in its strategy is as below:

$$\begin{aligned} &|\rho_j(a) - \rho_j(b)| \\ &= \left| \int_{x \in D_j(a_i, a_{-i})} \varrho_i(x, a) dx - \int_{x \in D_j(a'_i, a_{-i})} \varrho_i(x, b) dx \right|, \\ &= \left| \int_{x \in D_j(a) \cap \mathbf{1}_{i, j \in N_x}} \varrho_i(x, a) dx - \int_{x \in D_j(b) \cap \mathbf{1}_{i, j \in N_x}} \varrho_i(x, b) dx \right|. \end{aligned} \quad (47)$$

The equation (47) is valid because the change of CRE bias of BS  $i$  affect the load of BS  $j$  only at those locations where both BSs  $i$  and  $j$  can potentially serve the users.

$$|\rho_j(a) - \rho_j(b)| \leq \int_{x \in \mathcal{L}_{i,j} \in N_x} \varrho_i(x, a) dx + \int_{x \in \mathcal{L}_{i,j} \in N_x} \varrho_i(x, b) dx. \quad (48)$$

$$|\rho_j(a) - \rho_j(b)| \leq \nu_m \int_{x \in \mathcal{L}_{i,j} \in N_x} \frac{\lambda(x)}{\mu(x)} dx + \nu_m \int_{x \in \mathcal{L}_{i,j} \in N_x} \frac{\lambda(x)}{\mu(x)} dx. \quad (49)$$

$$|\rho_j(a) - \rho_j(b)| \leq 2\mu_m \nu_m \int_{x \in \mathcal{L}_{i,j} \in N_x} \lambda(x) dx. \quad (50)$$

$$\leq 2\mu_m \nu_m \varpi \int_{x \in \mathcal{L}_{i,j} \in N_x} \lambda(x) dx, \quad (51)$$

$$\leq 2\mu_m \nu_m \lambda_m \varpi, \quad (52)$$

The inequality (51) is obtained by using (14).

Let  $\rho_{\max}$  be the maximum load for a BS (assumed bounded away from 1). Consider function  $g^\alpha : x \rightarrow \frac{(1-x)^{1-\alpha}}{\alpha-1}$ . Its derivative is  $(1-x)^{-\alpha}$ . If  $0 < x < \rho_{\max}$ , then  $f$  is Lipschitz with constant  $(1-\rho_{\max})^{-\alpha}$ . This implies that for any  $x$  and  $y$ ,

$$|g^\alpha(x) - g^\alpha(y)| \leq (1-\rho_{\max})^{-\alpha} |x - y|.$$

Then from (35) we have

$$\xi_i^\varpi(a, b) = \sum_{j \in N_i^0 \setminus N_i^\varpi} g_j^\alpha(a_i, a_{-i}) - g_j^\alpha(a'_i, a_{-i}),$$

where  $g_j^\alpha(a_i, a_{-i}) = \frac{(1-\rho_j(a_i, a_{-i}))^{1-\alpha}}{\alpha-1}$ .

All in all, we obtain

$$\begin{aligned} |\xi_i(a, b)| &= \left| \sum_{j \in N_i^0 \setminus N_i^\varpi} \frac{(1-\rho_j(a))^{1-\alpha}}{\alpha-1} - \frac{(1-\rho_j(b))^{1-\alpha}}{\alpha-1} \right|, \\ &\leq \sum_{j \in N_i^0 \setminus N_i^\varpi} \left| \frac{(1-\rho_j(a))^{1-\alpha}}{\alpha-1} - \frac{(1-\rho_j(b))^{1-\alpha}}{\alpha-1} \right|, \\ &\leq |\mathcal{S}| (1-\rho_{\max})^{-\alpha} |\rho_j(a) - \rho_j(b)|, \\ &\leq |\mathcal{S}| (1-\rho_{\max})^{-\alpha} 2\mu_m \lambda_m \nu_m \varpi. \end{aligned} \quad (53)$$

The constraint in Theorem 3 is satisfied if  $|\xi| < \frac{\epsilon}{2(|\mathcal{X}|-1)}$ . Using the above upper bound of  $\xi$  we get the sufficient condition in (19). ■

## H. Neighborhood of macro BS

We show here that a similar neighborhood can be defined for macro BSs. Let  $\mathcal{L}_j$  be a set of all possible locations that can be served by BS  $j$ . It is defined as below:

$$\mathcal{L}_j = \{x \in \mathcal{L} : j \in N_x\},$$

where  $N_x$  is given by (13). We consider BS  $j$  as a neighbour to macro BS  $i$  if the load of BS  $j$  is significantly effected by the interference of macro BS  $i$ . Let denote a set of locations  $J_i = \{x \in \mathcal{L} : \max_{\{\theta_i, f\}} P_i^f(\theta_i) g_i(x) \geq I^{\text{th}}\}$  and  $J_i^c$  is its complement set. Formally, we define the neighborhood of macro BS  $i$  as:

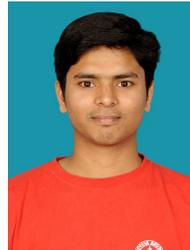
$$N_i^\varphi = \left\{ j \in \mathcal{S} : \frac{\int_{x \in \mathcal{L}_j \cap J_i} \lambda(x) dx}{\int_{x \in \mathcal{L}_j} \lambda(x) dx} \geq \varphi \right\}. \quad (54)$$

According to the above equation, BS  $j$  is a neighbour of macro BS  $i$  if macro BS  $i$  interference exceeds a threshold  $I^{\text{th}}$  at more than  $\varphi$  proportion of locations that can be served by BS  $j$ . A similar idea for defining neighborhood based on interference is proposed in [45]. Note that the threshold  $I^{\text{th}}$  takes into account the effect of ABS on SINR in (3) and load in (6). Particularly, if  $\max_{\{\theta_i, f\}} P_i^f(\theta_i) g_i(x) < I^{\text{th}}$  then the change in average data rate of BS  $j$  is bounded and we can come up with a similar condition for macro BSs as in Corollary 3 for the convergence of algorithms to optimal PNE.

## REFERENCES

- [1] M. S. Ali, P. Coucheny, and M. Coupechoux, "Load balancing in heterogeneous networks based on distributed learning in potential games," in *International Symp. Modeling and Optimisation in Mobile, Ad Hoc, and Wireless Netw.*, 2015.
- [2] J. Andrews, S. Singh, Q. Ye, X. Lin, and H. Dhillon, "An overview of load balancing in hetnets: old myths and open problems," *IEEE Wireless Commun.*, vol. 21, pp. 18–25, April 2014.
- [3] O. K. Tonguz and E. Yanmaz, "The mathematical theory of dynamic load balancing in cellular networks," *IEEE Trans. Mobile Comput.*, vol. 7, no. 12, pp. 1504–1518, 2008.
- [4] K. Son, S. Chong, and G. Veciana, "Dynamic association for load balancing and interference avoidance in multi-cell networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 7, pp. 3566–3576, 2009.
- [5] B. Rengarajan and G. De Veciana, "Architecture and abstractions for environment and traffic-aware system-level coordination of wireless networks," *IEEE Trans. Netw. ACM*, vol. 19, no. 3, pp. 721–734, 2011.
- [6] E. Stevens-Navarro, Y. Lin, and V. W. Wong, "An MDP-based vertical handoff decision algorithm for heterogeneous wireless networks," *IEEE Trans. Technol.*, vol. 57, no. 2, pp. 1243–1254, 2008.
- [7] S.-E. Elayoubi, E. Altman, M. Haddad, and Z. Altman, "A hybrid decision approach for the association problem in heterogeneous networks," in *Proc. INFOCOM*, pp. 1–5, 2010.
- [8] H. Kim, G. de Veciana, X. Yang, and M. Venkatachalam, "Distributed alpha-optimal user association and cell load balancing in wireless networks," *IEEE/ACM Trans. Netw.*, vol. 20, pp. 177–190, Feb 2012.
- [9] F. Xu, C. C. Tan, Q. Li, G. Yan, and J. Wu, "Designing a practical access point association protocol," in *Proc. INFOCOM*, pp. 1–9, 2010.
- [10] D. Niyato and E. Hossain, "Dynamics of network selection in heterogeneous wireless networks: An evolutionary game approach," *IEEE Trans. Veh. Technol.*, vol. 58, pp. 2008–2017, May 2009.
- [11] E. Aryafar, A. Keshavarz-Haddad, M. Wang, and M. Chiang, "RAT selection games in HetNets," in *Proc. INFOCOM*, pp. 998–1006, 2013.
- [12] T. Kudo and T. Ohtsuki, "Cell range expansion using distributed Q-learning in heterogeneous networks," *EURASIP J. Wireless Commun. Netw.*, pp. 1–10, Mar. 2013.
- [13] K. Lee, S. Kim, S. Lee, and J. Ma, "Load balancing with transmission power control in femtocell networks," in *Advanced Communication Technology (ICACT)*, pp. 519–522, Feb 2011.
- [14] A. Damnjanovic, J. Montojo, Y. Wei, T. Ji, T. Luo, M. Vajapeyam, T. Yoo, O. Song, and D. Malladi, "A survey on 3GPP heterogeneous networks," *IEEE Wireless Commun.*, vol. 18, no. 3, pp. 10–21, 2011.
- [15] J. Oh and Y. Han, "Cell selection for range expansion with almost blank subframe in heterogeneous networks," in *Proc. PIMRC*, pp. 653–657, Sept 2012.
- [16] K. Kitagawa, T. Komine, T. Yamamoto, and S. Konishi, "Performance evaluation of handover in LTE-advanced systems with pico cell range expansion," in *Proc. PIMRC*, pp. 1071–1076, Sep. 2012.

- [17] P. Ökqvist and A. Simonsson, "LTE HetNet trial - range expansion including micro/pico indoor coverage survey," in *Proc. VTC (Fall)*, pp. 1–5, Sep 2012.
- [18] S. Vasudevan, R. N. Pupala, and K. Sivanesan, "Dynamic eICIC-A proactive strategy for improving spectral efficiencies of heterogeneous LTE cellular networks by leveraging user mobility and traffic dynamics," *IEEE Trans. Wireless Commun.*, vol. 12, no. 10, pp. 4956–4969, 2013.
- [19] J. Wang, J. Liu, D. Wang, J. Pang, and G. Shen, "Optimized fairness cell selection for 3GPP LTE-A macro-pico HetNets," in *Proc. VTC (Fall)*, pp. 1–5, Sep 2011.
- [20] I. Siomina and D. Yuan, "Load balancing in heterogeneous LTE: Range optimization via cell offset and load-coupling characterization," in *Proc. ICC*, pp. 1357–1361, June 2012.
- [21] Y. Khan, B. Sayrac, and E. Moulines, "Surrogate based centralised son: Application to interference mitigation in LTE-A hetnets," in *Proc. VTC (Spring)*, pp. 1–5, 2013.
- [22] S. Deb, P. Monogioudis, J. Miernik, and J. P. Seymour, "Algorithms for enhanced inter-cell interference coordination (eICIC) in LTE hetnets," *IEEE/ACM Trans. Netw. (TON)*, vol. 22, no. 1, pp. 137–150, 2014.
- [23] A. Bedekar and R. Agrawal, "Optimal muting and load balancing for eicic," in *International Symp. Modeling and Optimisation in Mobile, Ad Hoc, and Wireless Netw.*, pp. 280–287, IEEE, 2013.
- [24] K. Kikuchi and H. Otsuka, "Proposal of adaptive control CRE in heterogeneous networks," in *Proc. PIMRC*, pp. 910–914, Sep 2012.
- [25] M. Al-Rawi, "A dynamic approach for cell range expansion in interference coordinated LTE-advanced heterogeneous networks," in *Proc. ICCS*, pp. 533–537, Nov 2012.
- [26] A. Tall, Z. Altman, and E. Altman, "Self organizing strategies for enhanced ICIC (eICIC)," in *International Symp. Modeling and Optimisation in Mobile, Ad Hoc, and Wireless Netw.*, pp. 318–325, IEEE, 2014.
- [27] Q. Ye, B. Rong, Y. Chen, M. Al-Shalash, C. Caramanis, and J. Andrews, "User association for load balancing in heterogeneous cellular networks," *IEEE Trans. Wireless Commun.*, vol. 12, pp. 2706–2716, June 2013.
- [28] Y. Liu, C. S. Chen, and C. W. Sung, "Joint optimization on inter-cell interference management and user attachment in lte-a hetnets," in *International Workshop on Resource Allocation, Cooperation and Competition in Wireless Networks (RAWNET)*, IEEE, 2015.
- [29] O. Candogan, A. Ozdaglar, and P. A. Parrilo, "Learning in near-potential games," in *IEEE Conf. Decision Control and European Control Conference*, pp. 2428–2433, 2011.
- [30] J. Mo and J. Walrand, "Fair end-to-end window-based congestion control," *IEEE/ACM Trans. Netw.*, vol. 8, no. 5, pp. 556–567, 2000.
- [31] B. S. Pradelski and H. P. Young, "Learning efficient nash equilibria in distributed systems," *Games and Economic behavior*, vol. 75, no. 2, pp. 882–897, 2012.
- [32] S. Singh, X. Zhang, and J. Andrews, "Joint rate and sinr coverage analysis for decoupled uplink-downlink biased cell associations in hetnets," *IEEE Trans. Wireless Commun.*, vol. 14, pp. 5360–5373, Oct 2015.
- [33] A. Goldsmith, *Wireless communications*. Cambridge university press, 2005.
- [34] M. Gudmundson, "Correlation model for shadow fading in mobile radio systems," *Electron. Lett.*, vol. 27, no. 23, pp. 2145–2146, 1991.
- [35] R. Fraile, J. F. Monserrat, J. Gozálviz, and N. Cardona, "Mobile radio bi-dimensional large-scale fading modelling with site-to-site cross-correlation," *European trans. on telecommunications*, vol. 19, no. 1, pp. 101–106, 2008.
- [36] R. Madan, J. Borran, A. Sampath, N. Bhushan, A. Khandekar, and T. Ji, "Cell association and interference coordination in heterogeneous LTE-A cellular networks," *IEEE J. Sel. Areas Commun.*, vol. 28, pp. 1479–1489, Dec. 2010.
- [37] K. Okino, T. Nakayama, C. Yamazaki, H. Sato, and Y. Kusano, "Pico cell range expansion with interference mitigation toward lte-advanced heterogeneous networks," in *ICC Workshop*, pp. 1–5, June 2011.
- [38] M. Shirakabe, A. Morimoto, and N. Miki, "Performance evaluation of inter-cell interference coordination and cell range expansion in heterogeneous networks for LTE-advanced downlink," in *ISWCS*, pp. 844–848, Nov. 2011.
- [39] M. Vajapeyam, A. Damnjanovic, J. Montojo, T. Ji, Y. Wei, and D. Mal-ladi, "Downlink FTP performance of heterogeneous networks for LTE-advanced," in *ICC Workshop*, pp. 1–5, June 2011.
- [40] I. Guvenc, "Capacity and fairness analysis of heterogeneous networks with range expansion and interference coordination," *IEEE Commun. Lett.*, vol. 15, pp. 1084–1087, Oct. 2011.
- [41] A. Morimoto, N. Miki, H. Ishii, and D. Nishikawa, "Investigation on transmission power control in heterogeneous network employing cell range expansion for LTE-advanced uplink," in *European Wireless Conf.*, pp. 1–6, Apr. 2012.
- [42] M. Eguizabal and A. Hernandez, "Interference management and cell range expansion analysis for LTE picocell deployments," in *Proc. PIMRC*, pp. 1592–1597, Sept 2013.
- [43] T. Bonald and L. Massoulié, "Impact of fairness on internet performance," in *ACM SIGMETRICS Performance Evaluation Review*, vol. 29, pp. 82–91, 2001.
- [44] D. Monderer and L. S. Shapley, "Potential Games," *Games and Economic Behavior*, vol. 14, pp. 124–143, May 1996.
- [45] Y. Xu, J. Wang, Q. Wu, A. Anpalagan, and Y.-D. Yao, "Opportunistic spectrum access in cognitive radio networks: Global optimization using local interaction games," *Selected Topics in Signal Processing, IEEE Journal of*, vol. 6, pp. 180–194, April 2012.
- [46] K. Yamamoto, "A comprehensive survey of potential game approaches to wireless networks," *IEICE Transactions on Communications*, vol. 98, no. 9, pp. 1804–1823, 2015.
- [47] J. R. Marden and J. S. Shamma, "Revisiting log-linear learning: Asynchrony, completeness and a payoff-based implementation," *Games and Economic Behaviour*, vol. 75, pp. 788–808, July 2012.
- [48] 3GPP, "Evolved Universal Terrestrial Radio Access (E-UTRA) and Evolved Universal Terrestrial Radio Access (E-UTRAN); Overall description; Stage 2," TS 36.300, 3GPP, Sept. 2008.
- [49] 3GPP, "Technical specification group radio access network; evolved universal terrestrial radio access (utra); further advancements for e-utra physical layer aspects," TR 36.814, 3GPP, Mar. 2010.



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