Optimal Distributed Channel Assignment in D2D Networks Using Learning in Noisy Potential Games

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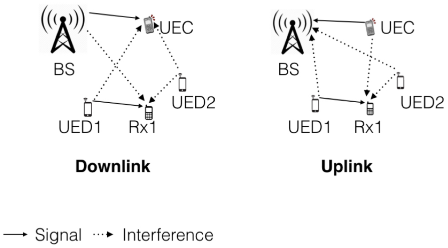
D2D Channel Assignement I

- Device-to-Device (D2D) networks increase the utilization of the spectrum by providing spatial reuse [Asadi14].
- A crucial problem in underlay D2D networks is to assign channels to UEs to increase its utilization while maintaining low interference.
- Specific D2D issue : lack of Channel State Information at BS on top of CSI estimation errors due to varying channel, delay, quantization, etc.



D2D Channel Assignement II

Related work :

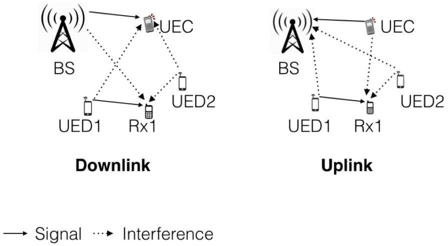
- Channel Assignment Problem (CAP) in wireless networks is NP-hard [Garay02].
- Various approaches : DP [Wang16], heuristics on graphs [Maghsudi16], game theory [Song14], linear or non-linear programming, Markov random fields [Ahmed19].
- Our approach : distributed, optimal, valid for any channel distribution, assumes imperfect channel estimation, with theoretical guarantees.
- A new framework : noisy potential games.
- Paper available on ArXiv (same title).

Outlines

- Network Model
- Problem Formulation
- Noisy Potential Games
- Learning in Presence of Noise
- Simulation Results

Network Model

- UECs are assigned orthogonal channels, UEDs reuse these channels.
- Every UE transmits on a single channel.
- Channel model : path-loss, shadowing and fast fading
- SINR of user i on channel c is denoted $\gamma_i(c)$.
- Average data rate is $\nu_i(c)$, estimated data rate is $\hat{\nu}_i(c)$.



Problem Formulation I

- Find an optimal channel assignment

$$\bar{c}^* \in \underset{\bar{c} \in \mathcal{F}^{|\mathcal{D}|}}{\operatorname{argmax}} \ \phi(\bar{c}),$$

where $\phi(\bar{c}) = \mathbb{E}[\hat{\phi}(\bar{c})]$ is the expected sum data rate,
and $\hat{\phi}(\bar{c}) = \sum_{j \in \mathcal{D}} \hat{\nu}_j(\bar{c})$ is the estimated sum data rate.

- $\bar{c} = (c_i, c_{-i})$ denotes a channel assignment vector where UE i is assigned the channel $c_i \in \mathcal{F}$ and UEs other than UE i are assigned the channel vector $c_{-i} \in \mathcal{F}^{|\mathcal{D}|-1}$.
- We seek to maximize the average sum data rate by using only estimates of these data rates.
- The above problem is a Stochastic Optimization Problem.

Problem Formulation II

Looking for a distributed solution, we define :

Definition

A CAP game is defined by the tuple $\hat{\mathcal{G}} := \{\mathcal{D}, \{X_i\}_{i\in\mathcal{D}}, \{\hat{U}_i\}_{i\in\mathcal{D}}\}$, where $\mathcal{D}$ is a set of UEs that are players of the game, $\{X_i\}_{i\in\mathcal{D}}$ are action sets consisting of orthogonal channels, $\hat{U}_i : X \to \mathcal{R}$ are random utility functions with finite expectation, and $X := X_1 \times X_2 \times \ldots X_{|\mathcal{D}|}$.

Noisy Potential Games I

In general, potential games are an attractive framework for distributed optimization, as soon as potential and objective functions are aligned :

- Global maximizers of the objective are optimal Nash Equilibria.
- NEs are maximizers of the potential.
- In finite games, NEs always exist.
- There are known algorithms that converges to optimal NEs.

Definition

A game $\mathcal{G} := \{\mathcal{D}, \{X_i\}_{i\in\mathcal{D}}, \{U_i\}_{i\in\mathcal{D}}\}$ is a (deterministic) potential game if there is a potential function $h : X \to \mathcal{R}$ such that $\forall i \in \mathcal{D}$, $\forall a_i, a_i' \in X_i$ and $\forall a_{-i} \in X_{-i}$,

$$U_i(a_i, a_{-i}) - U_i(a_i', a_{-i}) = h(a_i, a_{-i}) - h(a_i', a_{-i}). \tag{1}$$

Noisy Potential Games II

Because our CAP game has random utilities, we define noisy potential games :

Definition

Let the expected utility of player i be denoted as $U_i = \mathbb{E}[\hat{U}_i]$. The game $\hat{\mathcal{G}} := \{\mathcal{D}, \{X_i\}_{i\in\mathcal{D}}, \{\hat{U}_i\}_{i\in\mathcal{D}}\}$ is a noisy potential game if the game $\mathcal{G} := \{\mathcal{D}, \{X_i\}_{i\in\mathcal{D}}, \{U_i\}_{i\in\mathcal{D}}\}$ is a potential game.

Noisy Potential Games III

Utility design :

- Let $\hat{U}_{i,k}$ be the marginal contribution of player i to the global utility :

$$\hat{U}_{i,k}(a_i, a_{-i}) = \sum_{j \in \mathcal{D}(\mathbf{a_i})} \hat{\nu}_j(a_i, a_{-i}) - \sum_{j \in \mathcal{D}(\mathbf{a_i}) \setminus i} \hat{\nu}_j(a_i, a_{-i}), \quad (2)$$

- And $\hat{U}_i^N$ this contribution averaged over N samples :

$$\hat{U}_i^N(a_i, a_{-i}) = \frac{1}{N} \sum_{k=1}^{N} \hat{U}_{i,k}(a_i, a_{-i}), \quad (3)$$

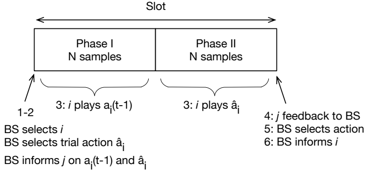
Proposition

A CAP game $\hat{\mathcal{G}}^N := \{\mathcal{D}, \{X_i\}_{i \in \mathcal{D}}, \{\hat{U}_i^N\}_{i \in \mathcal{D}}\}$ with utilities defined in (3), (2) is a noisy potential game with potential function $\phi(a)$.

Learning in Presence of Noise I

Proposed scheme (based on Binary Log Linear Learning – BLLA) :

1. BS randomly selects a player i and a trial action $\hat{a}_i$.
2. BS informs all j on channels $\{a_i(t-1), \hat{a}_i\}$ to estimate their data rates and feedback this info to the BS.
3. Player i plays action $a_i(t-1)$ and $\hat{a}_i$ during Phase I and Phase II.
4. All players on $a_i(t-1)$ and $\hat{a}_i$ feedback to the BS their two estimates of their sampled mean data rates corresponding to Phases I and II.
5. BS calculates the utility of player i and selects an action from the set $\{a_i(t-1), \hat{a}_i\}$ w.p. $(1 + e^{\Delta_i^N/\tau})^{-1}$ where $\Delta_i^N = \hat{U}_i^N(a(t-1)) - \hat{U}_i^N(\hat{a}_i, a_{-i}(t-1))$.
6. BS informs player i with the selected action.



Learning in Presence of Noise II

Remarks :

- BLLA generates an irreducible Markov chain over the action space.
- As $\tau \to 0$, the stationary distribution concentrates on few states.
- States whose limit probability is strictly positive are called stochastically stable states (SSS).
- In exact deterministic potential games, the stochastically stable states of BLLA are the maximizers of the potential function [Marden10].
- Few works deal with estimation errors : In [Leslie11], noise is zero-mean and has finite variance, N is an exponential function of the maximum difference of payoffs of any player and τ ; Smooth BR is proposed in [Coucheney14] but the influence of noise variance and N are not studied ; [Candogan11, Ali16] use the notion of near-potential games but convergence is not guaranteed when maximum pairwise difference is too high.

Learning in Presence of Noise III

Theorem

The stochastically stable states of BLLA are the global maximizers of the potential function $\phi(a)$ if one of the following holds.

1. *The estimation noise is bounded in an interval of size ℓ and the number of estimation samples used are :*

$$N \geq \left(\log\left(\frac{4}{\xi}\right) + \frac{2}{\tau}\right) \frac{\ell^2}{2\left(1-\xi\right)^2 \tau^2}, \tag{4}$$

where $0 < \xi < 1$.

2. *The estimation noise is unbounded with finite mean and variance. Let $M(\theta)$ be moment generating function of noise. Assume that $M(\theta)$ is finite. Let $\theta^* = \operatorname{argmax}_\theta \theta\left(1-\xi\right)\tau - \log\left(M(\theta)\right)$. The number of samples used are :*

$$N \geq \frac{\log\left(\frac{4}{\xi}\right) + \frac{2}{\tau}}{\log\left(\frac{e^{\theta^*(1-\xi)\tau}}{M(\theta^*)}\right)}. \tag{5}$$

Learning in Presence of Noise IV

Example :

- Let the noise has standard normal probability distribution. Then the stochastically stable states of BLLA are the global maximizers of the potential function if

$$N \geq \frac{2 \log \left(\frac{4}{\xi}\right) + \frac{4}{\tau}}{\tau^2 (1-\xi)^2}. \tag{6}$$

Remarks :

- Bounded noise bound is obtained using Hoefding inequality
- Unbounded noise bound is obtained using Chernoff bound

Learning in Presence of Noise V

Remarks :

- The convergence is in probability for fixed τ.
- Almost sure convergence is obtained for decreasing τ in the following result, as in simulated annealing [Hajek88].

Theorem

Consider BLLA with a decreasing parameter $\tau(t) = 1/\log(1+t)$, and the number of samples $N(\tau)$ is given by previous Theorem. Then, BLLA converges with probability 1 to the global maximizer of the potential function.

Learning in Presence of Noise VI

Sketch of the proof for fixed τ :

1. BLLA induces a regular Perturbed Markov chain, where the unperturbed chain is induced by best response and resistance of a transition is defined as the cost of deviating from best response.
2. SSSs correspond to the roots of minimum resistance trees of the Perturbed Markov Chain.
3. These roots are maximizers of the potential function.
4. Provided that N is chosen according to the theorem, we show that resistance of a transition is the same as in the deterministic case. This is due to the fact that the resistance introduced by errors is small wrt the one introduced by the learning algorithm.

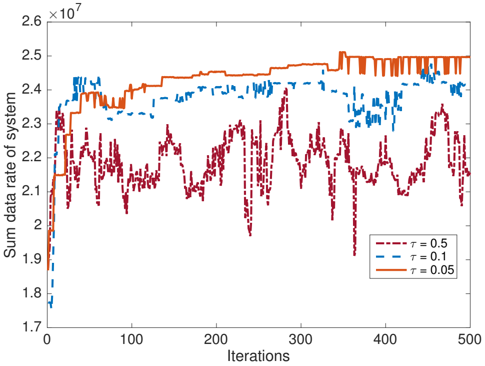
$\Rightarrow$ In the paper, we propose new easy generic rules to compute resistance of transitions.

Learning in Presence of Noise VII

Sketch of the proof for decreasing τ :

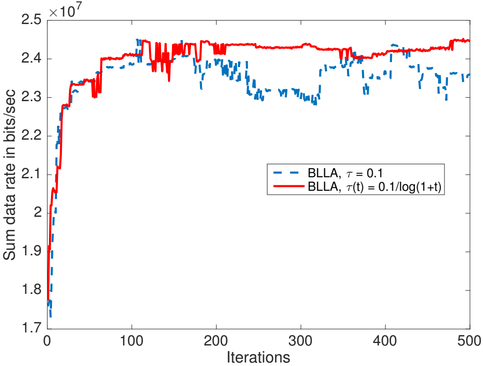
- We show that the algorithm generates a weakly ergodic non-homogeneous Markov chain. NB : a non-homogeneous Markov chain is weakly ergodic if the dependence of the state distribution on the initial state tends to zero when time tends to infinity.
- A result in [Anily87] ensures the convergence.

Simulation Results I



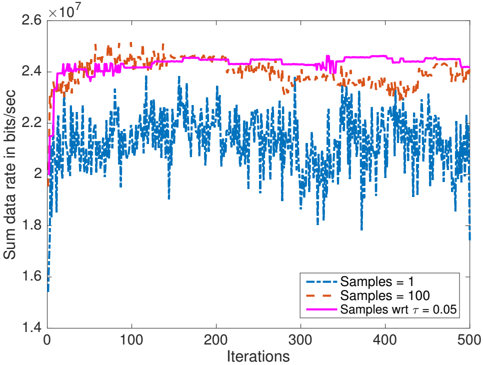
Higher temperatures exhibit bigger variations. The probability of being stuck for a long time in a local maximum increases with smaller temperatures.

Simulation Results II



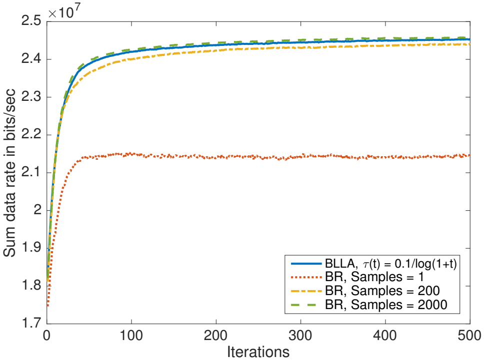
BLLA reaches the maximum with both fixed and decreasing temperatures. More variations for fixed temperature. For decreasing temperature, the probability of staying at the maximum is higher.

Simulation Results III



If N is chosen according to the Theorem, BLLA provides high and stable sum data rate.

Simulation Results IV



BR performs the worst when noise is ignored. As N increases BR and BLLA are similar. There is no theoretical guarantee for the convergence of BR with N finite.

Thank you for your attention !