Tabu search heuristic for competitive base station location problem.

Marceau Coupechoux, Ivan Davydov, Stefano Iellamo

1 System Model

We consider the following location problem. New 5G networks are deployed by two competitive operators (called resp. O_1 and O_2), which we refer to as a leader and a follower due to their sequential entering the market. They compete to serve clients by installing and configuring base stations (BS). We assume that the leader had already made a decision and operating a 5G network. Follower arrives at the market knowing the decision of the leader and set it's own 5G network. Follower is able to set up his BS on all the available cites. It is also possible to share the cite with the leader. In the latter case follower pays leader an additional sharing price. Each client choose the network considering the average quality of the service provided. The aim of the follower is to choose locations for his BS in order to maximize his profit.

1.1 Network and Propagation Model

Let $\mathscr{S}$ be the set of all sites, where base stations can be installed. This set is made of three subsets: $\mathscr{S} = \mathscr{S}_f \cup \mathscr{S}_1^o \cup \mathscr{S}_2^o$, where $\mathscr{S}_i^o$ is the set sites having a 4G base

Marceau Coupechoux
Télécom ParisTech and CNRS LTCI, 6, rue Barrault, Paris, France, e-mail: marceau.coupechoux@telecom-paristech.fr

Ivan Davydov
Sobolev Institute of Mathematics, Av.Koptyuga 4, Novosibirsk, Russia, e-mail: vann.davydov@gmail.com

Stefano Iellamo
Télécom ParisTech and CNRS LTCI, 6, rue Barrault, Paris, France, e-mail: iellamo@enst.fr

station installed by O_i, $i \in \{1,2\}$, and $\mathscr{S}_f$ is a set of free sites for potential new installations.

O_1 have a subset $\mathscr{S}_1^n$ of new 5G base stations installed. His new BSs are installed among the free sites and the sites having old BSs of O_1: $\mathscr{S}_1^n \subset \mathscr{S}_f \cup \mathscr{S}_1^o$. The rent price is set by O_1 for every site with its BS, i.e. in the set $\mathscr{S}_1^n \cup \mathscr{S}_1^o$. Now O_2 is deploying its 5G network by choosing a set $\mathscr{S}_2^n$ for its 5G BSs. He has a choice among all the sites in $\mathscr{S}$, i.e. $\mathscr{S}_2^n \subset \mathscr{S}$. If a 5G BSs of O_2 is placed on a site in $\mathscr{S}_1^n$, he will have to pay the sharing price fixed by O_1. Otherwise, he will have to pay for maintenance. At the end of this phase, some users leave O_1 and take a subscription with O_2.

Now let consider a user located at x. Let define the channel gain between location x and BS b as $g_b(x)$ and let assume that the transmit power of b is P_b. Signal to interference and noise ratio (SINR) of the considered user in x with respect to b is given by:

$$\gamma_b(x) = \frac{P_b g_b(x)}{\sum_{i \neq b} P_i g_i(x) + N}\ , \tag{1}$$

where N is the thermal noise power in the band. User in x is said to be *covered* by b if $\gamma_b(x) \geq \gamma_{min}$ for some threshold γ_{min}. User in x is said to be *served* by b if he is covered and $P_b g_b(x) \geq P_i g_i(x)$ for all $i \neq b$. Note that at every location, users can be served by at most one BS from each operator.

For a user located in x and served by station b, the physical data rate achievable by this user is denoted by $c_b(x)$, which is an increasing nonlinear function of $\gamma_b(x)$ with $c_b(x) = 0$ if $\gamma_b(x) < \gamma_{min}$.

1.2 Traffic Model

We assume there is a constant traffic demand in the network that operators will potentially serve. In every location x, there is a demand $\lambda(x)/\mu(x)$, where $\lambda(x)$ is the arrival rate and $1/\mu(x)$ is the average file size. Note that this demand in x is statistical and can be shared by O_1 and O_2 or not served at all. Let assume that x is covered by O_1 and a proportion $p_1(x)$ of the demand is served by BS b from O_1. A proportion $p_2 = 1 - p_1(x)$ of the demand is served by O_2. We focus in this paper on a specific case for p_1: If location x is not covered by O_1, $p_1(x) = 0$. Otherwise, p_1 does not depend on the location and depends only on the overall relative quality of service in the network O_1 compared to O_2. The idea behind this assumption is that users are mobile and they choose their operator not only with respect to the quality of service at a particular location but rather to the average experienced quality.

Then, the load created by x on b in the network of operator O_i is $p_i(x)\rho_{ib}(x)$, where $\rho_{ib}(x) = \frac{\lambda(x)}{\mu(x)c_{ib}(x)}$, where $c_{ib}(x)$ is the physical data rate in x and is an increasing function of $\gamma_{ib}(x)$. The index i is here to recall that the SINR, so the physical data rate, and the load are computed in the network of O_i. This is an important point because in the rest of the paper, station b is likely to be shared by both operators. We can now define the *load* of station b as: $p_i\rho_{ib}$, where $\rho_{ib} = \sum_{\mathscr{A}_{ib}} \rho_{ib}(x)$, where $\mathscr{A}_{ib}$ is the serving area of b, i.e., the set of locations served by b, in network O_i. BS b is stable if $p_i\rho_{ib} < 1$ and we will consider only scenarios where this condition is fulfilled.

Let us define by Λ, $\Lambda = \sum_{x\in\mathscr{A}} \lambda(x)$, is the total arrival rate in the network. The average throughput obtained by a random user from operator O_1 is given by:

$$t_i = \frac{1}{\Lambda} \sum_{b\in\mathscr{S}_i^n} \sum_{\mathscr{A}_{ib}} p_i \lambda(x)(1 - p_i\rho_{ib})c_{ib}(x). \tag{2}$$

We assume that users are the players of an evolutionary game. In this framework, the choice of a single user does not influence the average throughput of an operator. An equilibrium is reached when both average throughputs are the same. In this case, we equalize t_1 and t_2, which leads to the following quadratic equation:

$$\begin{aligned} f(p_1) = p_1^2 (&\sum_{b\in\mathscr{S}_2^n} \rho_{2b} \sum_{\mathscr{A}_{2b}} \lambda(y)c_{2b}(y) - \sum_{b\in\mathscr{S}_1^n} \rho_{1b} \sum_{\mathscr{A}_{1b}} \lambda(x)c_{1b}(x)) + \\ + p_1(&\sum_{b\in\mathscr{S}_1^n} \sum_{\mathscr{A}_{1b}} \lambda(x)c_{1b}(x) - 2 \sum_{b\in\mathscr{S}_2^n} \rho_{2b} \sum_{\mathscr{A}_{2b}} \lambda(y)c_{2b}(y) + \sum_{b\in\mathscr{S}_2^n} \sum_{\mathscr{A}_{2b}} \lambda(y)c_{2b}(y)) + \\ &+ \sum_{b\in\mathscr{S}_2^n} \rho_{2b} \sum_{\mathscr{A}_{2b}} \lambda(y)c_{2b}(y) - \sum_{b\in\mathscr{S}_2^n} \sum_{\mathscr{A}_{2b}} \lambda(y)c_{2b}(y) = 0. \end{aligned}$$

Let p_1^* be the operator 1 market share at equilibrium. Several cases arise:

- If $f(p_1) > 0$ for all $p_1 \in [0;1]$, then operator 1 is always preferred to operator 2, and $p_1^* = 1$.
- If $f(p_1) < 0$ for all $p_1 \in [0;1]$, then $p_1^* = 0$.
- if $f(p_1) = 0$ for some $p_1 \in [0;1]$, then there are one or several equilibrium points. In this case, we set $p_1^* = \max\{p_1 \in [0;1] : f(p_1) = 0\}$. The assumption behind this choice is that operator 1 has come first on the market. The dynamics of p_1 thus starts from 1 and decreases to the first encountered equilibrium point.

1.3 Pricing Model and Objective Function

There are operational costs that have to be paid regularly. These operational costs include traditional costs like electricity, maintenance, site renting, and possibly a sharing price. The sharing price is paid by O_2 to O_1 for every site where BSs are shared. Let λ be the traditional operational cost per unit of time for a single operator BS. Let $(1+\alpha)\lambda$ with $0 < \alpha < 1$ be the traditional operation cost for a shared BS. Let s_b the sharing price set by O_1 for its BS b. We assume that the revenues of an operator are proportional to the market share, i.e. $P_1 = p_1 * C$, where C is the total capacity of the market. The objective function (i.e., the profits) is revenues minus operational costs.

2 Problem Formulation

As mentioned in the introduction, we are interested in the problem of base station placement where one provider (follower) enters the market competing for the clients with already existing network (leader). Follower deployes his base stations on possible candidate sites so as to maximize his profit. In this section we present the mathematical model of this optimization problem. Let us introduce the decision variables: $y_j = 1$, if the follower install antenna on a site $j \in \mathscr{S}$, $y_j = 0$ otherwise. $y_{i,j} = 1$, if the location i is served from station $j, y_{i,j} = 0$ otherwise.

Now the competitive location problem can be written as a following mixed integer programming model:

$$\max_{y}((1-p_1)C - \sum_{j\in\mathscr{S}_1^o} s_j y_j - \sum_{j\in\mathscr{S}_f} s_j x_j y_j - \sum_{j\in\mathscr{S}_f\cup\mathscr{S}_2^o} \lambda y_j (1-x_j)) \tag{3}$$

subject to

$$P_b g_{ib} y_b \geq \gamma_{min} \sum_{j\in\mathscr{S}, j\neq b} P_j g_{ij} y_j + \gamma_{min} N - \Gamma(1-y_{ib}) \quad \forall b \in \mathscr{S}, i \in I \tag{4}$$

$$P_b g_{ib} y_b \geq P_j g_{ij} y_j - \Gamma(1-y_{ib}) \quad \forall b, j \in \mathscr{S}, i \in I \tag{5}$$

$$y_{ij} \leq y_j \quad \forall i \in I, j \in \mathscr{S} \tag{6}$$

The objective function (3) can be understood as the total profit obtained by the follower, computed as the difference between the expected revenue from clients served and the operational costs and sharing prices paid for the stations installed. The sharing payment gives additional profit to the leader, and reduces the gain of the follower. Constraints (4) are the SINR conditions for a location to be covered. When $y_{ib} = 1$, the expression boils down to the SINR condition with respect to the SINR threshold γ_{min}. Whenever $y_{ib} = 0$ then the condition is always fulfilled because of the large value of Γ. Constraints (5) combined with (4) state that the location satisfying the minimal SINR constraint is served by a BS providing the most powerful signal. Constraints (6) state that a service is possible only if a station is installed.

3 Tabu search approach

Although, the constraints of the problem are linear, due to realistic model of clients behavior it is not the case for the goal function. Latter fact makes it hard to apply a broad variety of approaches, which works well with linear integer programming problems. In order to tackle the follower problem we propose a tabu search heuristic framework, which performs well on similar problems [1].

The tabu search method has been proposed by Fred Glover. It is a so called trajectory metaheuristic and has been widely used to solve hard combinatorial optimization problems [2]. The method is based on the original local search scheme that lets one "travel" from one local optimum to another looking for a global one, avoiding local optimum traps. The main mechanism that allows it to get out of local optima is a tabu list, which contains a list of solutions from previous iterations which are prohibited to be visited on the subsequent steps. We use well-known Flip and Swap neighborhoods to explore the search space within y variables. Together with the tabu list, we exploit the idea of randomized neighborhoods. This feature allows to avoid looping, significantly reduces the time per iteration, and improves search efficiency. We denote by $Swap_q$ the part q of the $Swap$ neighborhood chosen at random. $Flip_q$ neighborhood is defined in the same way, but with the different value of parameter q.

Scheme of STS method:

1. Build an initial solution Y, define the randomization parameter q, initialize an empty tabu list.

2. Repeat until the stopping criterion is satisfied:

2.1. Construct neighborhoods $Flip_q(Y)$ and $Swap_q(Y)$ and remove forbidden elements from them.

2.2. If the neighborhood is empty, return to step 2.1, else find an adjacent solution Y' with the largest value of the follower's profit.

2.3. Let $Y = Y'$, update tabu list and if $Y' > Y^*$ update the record.

3. Show the best solution found Y^*.

The initial solution is chosen at random. The randomization parameters q for the neighborhoods are set to be sufficiently small. As the tabu list, we use an ordered list of units or pairs of the follower's facilities that have been closed and opened over the last few iterations. The length of the tabu list changes in a given interval during local search. If the best found solution begins to repeat itself, we increase the tabu list length by one; otherwise, we reduce it by one. The method stops after a given number of iterations or after a certain amount of computation time.

4 Experimental studies

The proposed approach have been implemented in C++ environment and tested on the randomly generated and real data instances. We generated 10 sets of instances with different number of client locations (20, 40, ..,200). All locations are chosen with the uniform distribution over the square area. The number of sites was 1/4 of number of clients locations. Leader occupies exactly half of the sites at random. All the other data was also generated at random. The aim of the experiment was to study the behavior and convergence of the approach. We run the algorithm on all the 100 instances, 10 runs per instance. Time limit was set to 5 sec. for each run. The algorithm has demonstrated strong convergence. Among all the instances there was only 3 examples, with different results on different runs. All of the examples was rather big (with 180, 200 and 200) locations. And the relative difference between outputs was less then 1%. The second experimental study concerns the real data. We use the client locations and base station cites of the part of 13-th arrondissement of Paris [3]. The geometric centers of the blocks are assumed to be client locations. We also use the coordinates of existing base stations in this area. We have run a number of experiments in order to testify the believability of the proposed model. The following table contains the results of the dependance of followers behavior from the sharing price, proposed by the leader. It can be seen from the table that high sharing price is not always the optimal one for the leader.

Table 1 Profit and market share of the follower

Sharing price	Leader share (p_1)	Follower profit	Leader profit	N shared sites	N opened sites
200	0.327	1755	1314	2	7
220	0.327	1715	1354	2	7
250	0.371	1633	1436	2	7
280	0.393	1589	1420	1	6
310	0.416	1574	1338	0	6

5 Conclusions

We have considered new competitive base stations location problem with sharing. We have proposed a mathematical model for this problem and tabu search based heuristic for obtaining good solutions rather fast. Computational results shows the believability of the model and allows to expand both the model and method on the bilevel problem.

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